THOMAStutorials

JEE (FINAL)

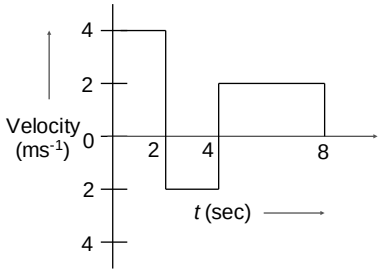
Date : **TEST NO :** 06

PCM

Time : 03 HRS **MARKS** : 360

Single Correct Answer Type

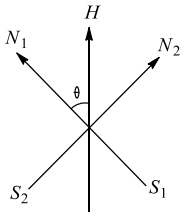
1. The time taken by an electron to go from ground state to excited state is one shake (one shake $= 10^{-8}$s). this time in nanosecond will be
   a) 10 ns b) 4 ns c) 2 ns d) 25 ns
2. A body is moving in a straight line a shown in velocity-time graph. The displacement and distance travelled by in 8s are respectively



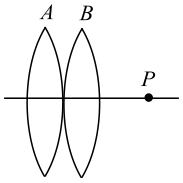
   a) 12 m, 20 m b) 20m, 12 m
   c) 12 m, 12 m d) 20 m, 20 m
3. A body executing uniform circular motion has at any instant its velocity vector and acceleration vector
   a) along the same direction b) in opposite direction
   c) normal to each other d) not related to each other
4. A man is standing at the centre of frictionless pond of ice. How can he get himself to the shore
   a) By throwing his shirt in vertically upward direction
   b) By spitting horizontally
   c) He will wait for the ice to melt in pond
   d) Unable to get at the shore
5. A body constrained to move in the y-direction is subjected to force $\mathbf{F} = 2\hat{\boldsymbol{\imath}} + 15\hat{\boldsymbol{\jmath}} + 6\hat{\boldsymbol{k}}$ N. The work done by this force in moving the body through a distance of 10 m along y-axis is
   a) 100 J b) 150 J c) 120 J d) 200 J
6. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is
   a) $2ml^2$ b) $\sqrt{3}\,ml^2$ c) $3\,ml^2$ d) ml^2
7. A planet has twice the radius but the mean density is $\frac{1}{4}$th as comparsed to earth.What is the ratio of escape velocity from earth to that from the planet?
   a) 3:1 b) 1:2 c) 1:1 d) 2:1
8. Two wires of the same length and same material but radii in the ratio of 1 : 2 are stretched by unequal forces to produce equal elongation. The ratio of the two forces is
   a) 1 : 1 b) 1 : 2 c) 2 : 3 d) 1 : 4
9. A square plate of 0.1 m side moves parallel to a second plate with a velocity of 0.1 m/s, both plates being immersed in water. If the viscous force is 0.002 N and the coefficient of viscosity is 0.01 poise, distance between the plates in m is
   a) 0.1 b) 0.05 c) 0.005 d) 0.0005
10. The coefficient of superficial expansion of a solid is 2×10^{-5}/°C. Its coefficient of linear expansion is
    a) 4×10^{-5}/°C b) 3×10^{-5}/°C
    c) 2×10^{-5}/°C d) 1×10^{-5}/°C
11. In a Carnot engine, the temperature of reservoir is 972°C and that of sink is 27°C. If the work done by the engine when it transfers heat from reservoir to sink is 12.6×10^6J, the quantity of heat absorbed y the engine from the reservoir is
    a) 16.8×10^6J b) 4×10^6J
    c) 7.6×10^6J d) 4.25×10^6J
12. If the internal energy of n_1 moles of He at temperature 10 T is equal to the internal energy of n_2 mole of hydrogen at temperature 6 T. the ratio of $\frac{n_1}{n_2}$ is
    a) $\frac{3}{5}$ b) 2 c) 1 d) $\frac{5}{3}$
13. Two simple harmonic motion are represented by
    $y_1 = 5\left(\sin 2\pi t + \sqrt{3}\cos 2\pi t\right)$
    $y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$
    The ratio of the amplitudes of two SHM's is
    a) 1 :1 b) 1 :2 c) 2 :1 d) $1 : \sqrt{3}$
14. In two similar wires of tension 16 N and T, 3 beats are heard, then T=

a) 49 N b) 25 N
c) 64 N d) None of these

15. Choose the incorrect statement from the following. When two identical capacitors are charged individually to different potentials and connected parallel to each other after disconnecting them from the source
    a) Net charge equals the sum of initial charges
    b) The net energy stored in the two capacitors is less than the sum of the initial individual energies
    c) The net potential difference across them is different from the sum of the individual initial potential difference
    d) The net potential difference across them equals the sum of the individual initial potential differences
16. Two points $P$and Q are maintained at the potentials of 10V and -4V respectively. The work done in moving 100 electrons from $P$to Q is
    a) -19×10^{-17}J b) 9.60×10^{-17}J
    c) -2.24×10^{-16}J d) 2.24×10^{-16}J
17. A cell of emf E and internal resistance r supplies currents for the same time t through external resistance $R_1 = 100\ \Omega$ and $R_2 = 40\ \Omega$ separately. If the heat developed in both the cases in the same, then the internal resistance of the cell is given by
    a) $28.6\ \Omega$ b) $70\ \Omega$ c) $63.3\ \Omega$ d) $140\ \Omega$
18. The masses of three wires of copper are in the ratio 1: 3: 5 and lengths are in the ratio 5: 3: 1. Then the ratio of their electrical resistances are
    a) 1: 3: 5 b) 5: 3: 1
    c) 1: 15: 25 d) 125: 15: 1
19. A uniform electric field and a uniform magnetic field are produced, pointing in the same direction. If an electron is projected with its velocity pointing in the same direction
    a) The electron will turn to its right
    b) The electron will turn to its left
    c) The electron velocity will increase in magnitude
    d) The electron velocity will decrease in magnitude
20. Two magnets of equal mass are joined at 90° each other as shown in figure. Magnet $N_1\ S_1$ has a magnetic moment $\sqrt{3}$ times that of $N_2\ S_2$. The arrangement is pivoted so that it is free to rotate in horizontal plane. When in equilibrium, what angle should $N_1\ S_1$ make with magnetic meridian?



    a) 75° b) 60° c) 30° d) 45°
21. A conducting rod of length l is falling with a velocity v perpendicular to a uniform horizontal magnetic field B. The potential difference between its two ends will be
    a) $2Blv$ b) Blv c) $\frac{1}{2}Blv$ d) $B^2l^2v^2$
22. An alternating e.m.f. is applied to purely capacitive circuit. The phase relation between e.m.f. and current flowing in the circuit is **or** In a circuit containing capacitance only
    a) e.m.f. is ahead of current by $\pi/2$
    b) Current is ahead of e.m.f. by $\pi/2$
    c) Current lags behind e.m.f. by π
    d) Current is ahead of e.m.f. by π
23. Which of the following shows green house effect?
    a) Ultraviolet rays b) Infrared rays
    c) X-rays d) None of these
24. Two convex lenses placed in contact form the image of a distant object at P. If the lens B is moved to the light, the image will



    a) Move to the left
    b) Move to the right
    c) Remain at P
    d) Move either to the left to right, depending upon focal length of the lenses
25. When light is incident on a diffraction grating, the zero order principal maximum will be
    a) Spectrum of the colours b) White
    c) One of the component colours d) Absent
26. The work function of aluminium is 4.2 eV. If two photons, each of energy 3.5 eV strike an electron of aluminium, then emission of electrons will be
    a) Possible

b) Not possible
c) Data is incomplete
d) Depends upon the density of the surface

27. The first excitation potential of a given atom is 10.2 V. Then ionisation potential must be
a) 20.4 V b) 13.6 V c) 30.6 V d) 40.8 V

28. Which one of the following statement is true, if half-life of a radioactive substance is 1 month?
a) 7/8th part of the substance will disintegrate in 3 months
b) 1/8th part of the substance will remain undecayed at the end of 4 months.
c) The substance will disintegrate completely in 4 months.
d) 1.16th part of the substance will remain undecayed at the end of 3 months

29. In a triode valve, the plate resistance is 10000 Ω and the anode load resistance is 30000 Ω. If the amplification factor is 36, then the voltage gain is
a) 9 b) 27 c) 36 d) 108

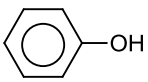
30. What is the range of the characteristic impedance of a coaxial cable?
a) Between 150 W to 600 W
b) Between 50 W to 70 W
c) Between 0 W to 50 W
d) Between 100 W to 150 W

31. For the reaction, $Fe_2O_3 + 3CO \rightarrow 2\,Fe + 3CO_2$ the volume of carbon monoxide required to reduce one mole of ferric oxide is
a) 22.4 dm^3 b) 44.8 dm^3 c) 67.2 dm^3 d) 11.2 dm^3

32. The de-Broglie wavelength of a tennis ball of mass 60g moving with a velocity of 10 m/s is approximately (Planck's constant, $h = 6.63 \times 10^{-34}$Js)
a) 10^{-33}m b) 10^{-31}m c) 10^{-16}m d) 10^{-25}m

33. In the Periodic Table metals usually used as catalyst belong to
a) f-block b) d-block
c) p-block d) s-block

34. Which of the following molecules/ ions does not contain unpaired electrons?
a) O_2^{2-} b) B_2 c) N_2^+ d) O_2

35. Which has maximum vapour pressure at a given temperature?
a) $CH_3CH_2CH_2OH$ b) 
c) $CH_3 - O - CH_3$ d) CH_3COOH

36. If the bond dissociation energies of XY, X_2 and Y_2 (all diatomic molecules) are in the ratio of 1: 1: 05 and ΔH for the formation of XY is -200 kJ mol^{-1}. The bond dissociation energy of X_2 will be
a) 100 kJ mol^{-1} b) 800 kJ mol^{-1}
c) 300 kJ mol^{-1} d) 400 kJ mol^{-1}

37. For the reaction, $C(s) + CO_2(g) \rightleftharpoons 2CO(g)$, the principle pressure of CO_2 and CO are 2.0 and 4.0 atm respectively at equilibrium. The K_p for the reaction is
a) 2.0 b) 4.0 c) 8.0 d) 1.6

38. The value of n in $MnO_4^- + 8H^+ + ne^- \rightarrow Mn^{2+} + 4H_2O$ is
a) 5 b) 4 c) 2 d) 3

39. Water is oxidised to oxygen by
a) ClO_2 b) $KMnO_4$ c) H_2O_2 d) Fluorine

40. Which one of the following electrolytes is used in Down's process of extracting sodium metal?
a) NaCl + KCl + KF b) NaCl
c) NaOH + KCl + KF d) NaCl + NaOH

41. Butter of tin is
a) $SnCl_2 \cdot 5H_2O$ b) $SnCl_2 \cdot 2H_2O$
c) $SnCl_4 \cdot 4H_2O$ d) $SnCl_4 \cdot 5H_2O$

42. The correct structure of 4-bromo-3-methyl-but-1-ene.
a) $Br - CH = C(CH_3)_2$
b) $CH_2 = CH - CH(CH_3) - CH_2Br$
c) $CH_2 = C(CH_3)CH_2CH_2Br$
d) $CH_3 - C(CH_3) = CHCH_2 - Br$

43. The addition of HCl to 3, 3, 3-trichloropropene gives
a) $Cl_3CCH_2CH_2Cl$ b) $Cl_3CCH_2CHCl_2$
c) $Cl_2CHCH_2CHCl_2$ d) $Cl_2CHCH(Cl)CH_2Cl$

44. A secondary pollutant is
a) CO b) CO_2
c) PAN d) Aerosol

45. Percentage of free space in cubic close packed structure and in body centred packed structure are respectively
a) 30% and 26% b) 26% and 32%
c) 32% and 48% d) 48% and 26%

46. Phenol dimerises in benzene having van't Hoff factor 0.54. What is the degree of association?
a) 1.92 b) 0.98 c) 1.08 d) 0.92

47. The charge required for reduction of 1 mole of $Cr_2O_7^{2-}$ ions to Cr^{3+} is
a) 96500 C b) 2 × 96500 C
c) 3 × 96500 C d) 6 × 96500 C

48. In the respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following

statements is correct?

a) K is equilibrium constant

b) A is adsorption factor

c) E_a is energy of activation

d) R is Rydberg constant

49. In which of the following reactions colloids are not prepared by the double decomposition method?

a) $2H_3AsO_4 + 3H_2S \rightarrow As_2S_3 + 6H_2O$

b) $3K_4[Fe(CN)_6] + 4FeCl_3 \rightarrow Fe_4[Fe(CN)_6]_3 + 12KCl$

c) $Mg(CN)_2 + H_2S \rightarrow HgS + 2HCN$

d) $Cu + HgCl_2 \rightarrow CuCl_2 + Hg$

50. van-Arker method of purification of metals involves converting the metal to a

a) Volatile stable compound

b) Non-volatile stable compound

c) Volatile unstable compound

d) None of the above

51. Welding of magnesium can be done in an atmosphere of

a) Xe b) He c) Kr d) Ne

52. In aqueous solution Eu^{2+} ion acts as

a) An oxidizing agent b) A reducing agent

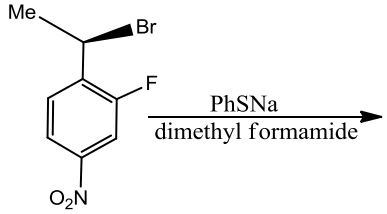
c) An acid d) All of these

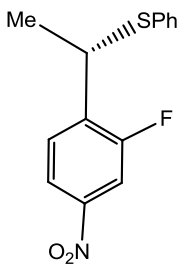
53. Which of the following will exhibit optical isomerism?

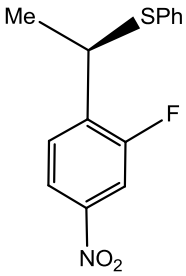
a) $[Cr(en)(H_2O)_4]^{3+}$ b) $[Cr(en)_3]^{3+}$

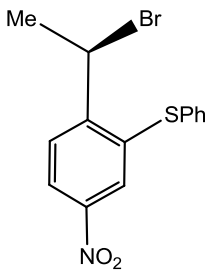
c) $trans$-$[Cr(en)_2Cl_2]^+$ d) $[Cr(NH_3)_6]^{3+}$

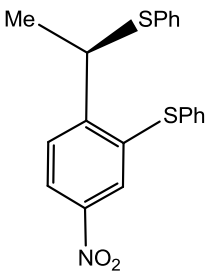
54. The major product of the following reaction is



a) 

b) 

c) 

d) 

55. Oxygen containing organic compound upon oxidation forms a carboxylic acid as the only organic product with its molecular mass higher by 14 units. The organic compound is

a) An aldehyde b) A primary alcohol

c) A secondary alcohol d) A ketone

56. Benzyl alcohol can be prepared from benzaldehyde by

a) Friedel-Craft's reaction

b) Cannizaro's reaction

c) Kolbe's reaction

d) Reimer-Tiemann reaction

57. Given the following sequence of reactions,

$$CH_3CH_2I \xrightarrow{NaCN} A \xrightarrow[\text{Partial hydrolysis}]{OH^-} B \xrightarrow{Br_2/NaOH} C$$

The major product 'C' is

a) $CH_3CH_2NH_2$ b) $CH_3.CH_2C(=O)NHBr$

c) $CH_3.CH_2 - COONH_4$ d) $CH_3.CH_2C(=O)NBr_2$

58. On fermentation, glucose yields

a) Ethanol b) Ethanal

c) Acetic acid d) Fructose

59. Natural rubber is polymer of

a) $H_2C = C(Cl) - CH = CH_2$

b) $H_2C = C(C_6H_5) - CH = CH_2$

c) $CH = CH_2$

d) $-(CH_2-CH_2)_n-$

60. Luminal, a barbiturate drug is used as a/an

a) Antihistamine b) Sedative

c) Antiseptic d) Antimalarial

61. If $n(A) = 4, n(B) = 3, n(A \times B \times C) = 240$, then $n(C)$ is equal to

a) 288 b) 1 c) 12 d) 2

62. Domain of definition of the function $f(x) = \sqrt{\sin^-(2x) + \frac{\pi}{6}}$ for real valued x, is

a) $\left[-\frac{1}{4}, \frac{1}{2}\right]$ b) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ c) $\left(-\frac{1}{2}, \frac{1}{9}\right)$ d) $\left[-\frac{1}{4}, \frac{1}{4}\right]$

63. If $\sin A = \frac{1}{\sqrt{10}}$ and $\sin B = \frac{1}{\sqrt{5}}$, where A and B are positive acute angles, then $A + B$ is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

64. Matrix A is such that $A^2 = 2A - I$ where I is the identity matrix, then for $n \geq 2, A^n$ is equal to
a) $n\,A - (n-1)I$ b) $n\,A - I$
c) $2^{n-1}A - (n-1)I$ d) $2^{n-1}A - I$

65. One of the square root of $6 + 4\sqrt{3}$ is
a) $\sqrt{3}(\sqrt{3}+1)$ b) $-\sqrt{3}(\sqrt{3}-1)$
c) $\sqrt{3}(-\sqrt{3}+1)$ d) None of these

66. The largest interval for which $x^{12} - x^9 + x^4 - x + 1 > 0$ is
a) $-4 < x < 0$ b) $0 < x < 1$
c) $-100 < x < 100$ d) $-\infty < x < \infty$

67. Consider the following statements :
1.The product of r consecutive natural numbers is always divisible by r.
2.The total number of proper positive divisors of 115500 is 94
3. A pack of 52 cards can be divided equally among four players order in $\frac{52!}{(13!)^4}$ ways.
Which of the statement given above is/are correct?
a) Only (1) b) Only (2)
c) Only (3) d) All of (1), (2) and (3)

68. If n is a positive integer, then $n^3 + 2n$ is divisible by
a) 2 b) 6 c) 15 d) 3

69. Let $a, p, q, r, s \in R \sim \{0\}$.
If $3a^2 + 2\left(\frac{1}{p} - \frac{1}{s}\right)a + \frac{1}{p^2} + \frac{1}{q^2} + \frac{1}{r^2} - 2\left(\frac{1}{pq} + \frac{1}{qr} + \frac{1}{rs}\right) \leq 0$ for some real a, then p, q, r, s are in
a) AP b) GP c) HP d) AGP

70. If the lines $x + 3y - 9 = 0, 4x + by - 2 = 0$ and $2x - y - 4 = 0$ are concurrent, then b equals
a) -5 b) 5 c) 1 d) 0

71. Equation of chord of an ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, whose mid point is $(1, 1)$, is
a) $25x + 9y = 36$
b) $9x + 25y = 34$
c) $9x - 25y = 34$
d) None of these

72. The value of $\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2}$ is
a) 0 b) 1 c) e^2 d) e^4

73. For any two statements p and q, $\sim(p \vee q) \vee (\sim p \wedge q)$is logically equivalent to
a) p b) $\sim p$ c) q d) $\sim q$

74. If in a frequency distribution, the mean and median are 21 and 22 respectively, then its mode is approximately
a) 24.0 b) 25.5 c) 20.5 d) 22.0

75. If three natural numbers from 1 to 100 are selected randomly, then probability that all are divisible by both 2 and 3, is
a) $\frac{4}{105}$ b) $\frac{4}{33}$ c) $\frac{4}{35}$ d) $\frac{4}{1155}$

76. A person observes the angle of elevation of a building as 30°. The persons proceeds towards the building with a speed of $25(\sqrt{3}-1)$ m/h. After two hours, he observes the angle of elevation as 45°.The height of the building (in metres)is
a) $50(\sqrt{3}-1)$ b) $50(\sqrt{3}+1)$
c) 50 d) 100

77. The angle of elevation of the sun, if the length of the shadow of a tower is $\sqrt{3}$ times the height of the pole, is
a) 150° b) 30° c) 60° d) 45°

78. Derivative of the function $f(x) = \log_5(\log_7 x), x > 7$ is
a) $\frac{1}{x(\log 5)(\log 7)(\log_7 x}$ b) $\frac{1}{x(\log 5)(\log 7)}$
c) $\frac{1}{x(\log x)}$ d) None of these

79. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to
a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) π d) $\frac{3\pi}{4}$

80. The system of equation
$3x - y + 4z = 3$
$x + 2y - 3z = -2$
$6x + 5y + \lambda z = -3$
Has at least one solution, if
a) $\lambda = -5$ b) $\lambda = 5$ c) $\lambda = 3$ d) $\lambda = -13$

81. The coefficient of x in $f(x) = \begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}, -1 < x \leq 1$, is
a) 1 b) -2 c) -1 d) 0

82. If $f(x) = |\log_e |x||$, then $f'(x)$ equals
a) $\frac{1}{|x|}, x \neq 0$
b) $\frac{1}{x}$ for $|x| > 1$ and $\frac{-1}{x}$ for $|x| < 1$
c) $\frac{-1}{x}$ for $|x| > 1$ and $\frac{1}{x}$ for $|x| < 1$

d) $\frac{1}{x}$ for $|x| > 0$ and $-\frac{1}{x}$ for $x < 0$

83. A particle is moving in a straight line such that the distance described 's' and the time taken 't' are given by $t = as^2 + bs + c, a > 0$. If v is the velocity of the particle at any time t, then acceleration is
a) $-2av$ b) $-2av^2$
c) $-2av^3$ d) None of these

84. $\int\{f(x)g''(x) - f''(x)g(x)\}dx$ is equal to
a) $\frac{f(x)}{g'(x)} + c$
b) $f'(x)g(x) - f(x)g'(x) + c$
c) $f(x)g'(x) - f'(x)g(x) + c$
d) $f(x)g'(x) + f'(x)g(x) + c$

85. The value of $\int_0^{\pi/2} \frac{dx}{1+\cot x}$ is
a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

86. The volume of the solid generated by the revolving of the curve $y = \frac{a^3}{a^2 + x^2}$ about $x-$axis is
a) $\frac{1}{2}\pi^3 a^2$ cu units b) $\pi^3 a^2$ cu units
c) $\frac{1}{2}\pi^2 a^3$ cu units d) $\pi^2 a^3$ cu units

87. The equation of family of a curve is $y^2 = 4a(x + a)$ then differential equation of the family is
a) $y = y' + x$ b) $y = y'' + x$
c) $y = 2y'x + yy'^2$ d) $y'' + y' + y^2 = 0$

88. Le the vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ be such that $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}}) = 0$. Let P_1 and P_2 be planes determined by pair of vectors $\vec{\mathbf{a}}, \vec{\mathbf{b}}$ and $\vec{\mathbf{c}}, \vec{\mathbf{d}}$ respectively. Then, the angle between P_1 and P_2 is
a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

89. If $(2, -1, 3)$ is the foot of the perpendicular drown from the origin to the plane, then the equation of the plane is
a) $2x + y - 3z + 6 = 0$ b) $2x - y + 3z - 14 = 0$
c) $2x - y + 3z - 13 = 0$ d) $2x + y + 3z - 10 = 0$

90. A wholesale merchant wants to start the business of cereal with Rs 24000. Wheat is Rs 400 per quintal and rice is Rs 600 per quintal. He has capacity to store 200 quintal cereal. He earns the profit Rs 25 per quintal on wheat and Rs 40 per quintal on rice. If he stores x quintal rice and y quintal wheat, then for maximum profit the objective function is
a) $25x + 40y$ b) $40x + 25y$
c) $400x + 600y$ d) $\frac{400}{40}x + \frac{600}{25}y$

THOMAStutorials

Date : **JEE (FINAL)** **TEST NO** : 06
TIME : 3 HRS **PCM** **MARKS :** 360

: ANSWER KEY :

1)	**a**	**2)**	**a**	**3)**	**c**	**4)**	**b**	**5)**	**b**	**6)**	**c**	**7)**	**c**
8)	**d**	**9)**	**d**	**10)**	**d**	**11)**	**a**	**12)**	**c**	**13)**	**c**	**14)**	**a**
15)	**d**	**16)**	**d**	**17)**	**c**	**18)**	**d**	**19)**	**d**	**20)**	**c**	**21)**	**b**
22)	**b**	**23)**	**b**	**24)**	**b**	**25)**	**b**	**26)**	**b**	**27)**	**b**	**28)**	**a**
29)	**b**	**30)**	**b**	**31)**	**c**	**32)**	**a**	**33)**	**b**	**34)**	**a**	**35)**	**c**
36)	**b**	**37)**	**c**	**38)**	**a**	**39)**	**d**	**40)**	**a**	**41)**	**d**	**42)**	**b**
43)	**a**	**44)**	**c**	**45)**	**b**	**46)**	**d**	**47)**	**b**	**48)**	**c**	**49)**	**d**
50)	**a**	**51)**	**b**	**52)**	**b**	**53)**	**b**	**54)**	**a**	**55)**	**b**	**56)**	**b**
57)	**a**	**58)**	**a**	**59)**	**a**	**60)**	**b**	**61)**	**d**	**62)**	**a**	**63)**	**d**
64)	**a**	**65)**	**d**	**66)**	**d**	**67)**	**d**	**68)**	**d**	**69)**	**c**	**70)**	**a**
71)	**b**	**72)**	**c**	**73)**	**b**	**74)**	**a**	**75)**	**d**	**76)**	**c**	**77)**	**b**
78)	**a**	**79)**	**a**	**80)**	**a**	**81)**	**b**	**82)**	**b**	**83)**	**c**	**84)**	**c**
85)	**d**	**86)**	**c**	**87)**	**c**	**88)**	**a**	**89)**	**b**	**90)**	**b**		

: HINTS AND SOLUTIONS :

1 **(a)**

$n_1u_1 = n_2u_2$

$n_2 = \dfrac{1 \text{ shake}}{1 \text{ ns}}$

$= \dfrac{10^{-8}\text{s}}{10^{-9}\text{s}}$

$\therefore \quad n_2 = 10$

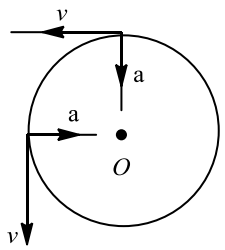
2 **(a)**

Displacement $= (2 \times 4 - 2 \times 2 + 2 \times 4) = 12\text{m}$

$= 2 \times 4 + 2 \times 2 + 2 \times 4 = 20\text{m}$

3 **(c)**

An object moving in uniform circular motion is moving around the perimeter of the circle with a constant speed. While the speed of object is constant, its velocity is changing. Velocity being a vector quantity has a constant magnitude but a changing direction. The direction is always directed tangent line is always pointing in a new direction. Also when it is moving in circular motion towards the centre, hence acceleration is perpendicular to velocity.



5 **(b)**

Force $\text{F} = (2\hat{\text{I}} + 15\hat{\text{j}} + 6\hat{\text{k}})\text{N}$

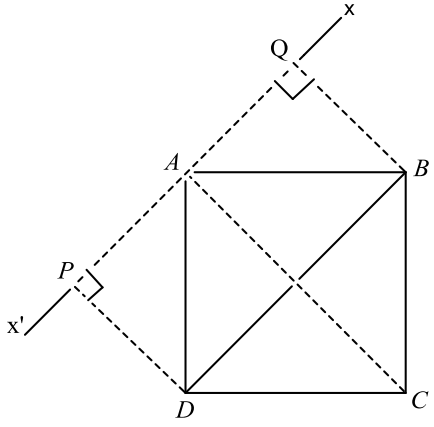
Displacement $\text{s} = 10\hat{\text{j}}\text{m}$

$W = \text{F} \cdot s = (2\hat{\text{I}} + 15\hat{\text{j}} + 6\hat{\text{k}}) \cdot (10\hat{\text{j}}) = 150\text{J}$

6 **(c)**

The situation is shown in figure

$$I_{XX''} = m \times DP^2 + m \times BQ^2 + m \times CA^2$$



$$= m \times 2 \times \left(\frac{\sqrt{2}l}{2}\right)^2 + m \times \left(\sqrt{2}l\right)^2$$
$$= 3ml^2$$

7 **(c)**

The maximum velocity with which a body must be projected in the atmosphere, so as to enable it to just overcome the gravitational pull, is known as escape velocity.

Escape velocity from earth's surface is

$$v_{\text{es}} = \sqrt{\frac{2GM_e}{R_e}}$$
$$= \sqrt{\frac{2G \cdot \frac{4}{3}\pi R_e^3 d_e}{R_e}} \qquad \left(\because M = \frac{4}{3}\pi R_e^3 d_e\right)$$

or $v_{\text{es}} \propto \sqrt{d_e} \times R_e$ ....(i)

similarly, for a planet

$$v'_{\text{es}} \propto \sqrt{d_p} \times R_p \qquad \text{...(ii)}$$

So, $\frac{v_{\text{es}}}{v'_{es}} = \left(\frac{d_e}{d_p}\right)^{1/2} \times \frac{R_e}{R_p}$

Given, $d_p = \frac{1}{4}d_e, R_p = 2R_e$

$$\frac{v_{\text{es}}}{v_{\text{es}}} = \left(\frac{\frac{d_e}{d_e}}{\frac{4}{21}}\right)^{\frac{1}{21}} \times \frac{R_e}{2R_e}$$
$$= (4)^{1/2} \times \frac{1}{2}$$
$$= 2 \times \frac{1}{2} = 1$$

So, $\frac{v_{\text{es}}}{v'_{\text{es}}} = 1:1$

8 **(d)**

$Y = \frac{Fl}{A\Delta l}$

In the given problem, Y, l and Δl are constants .

$\therefore\ F \propto A$

Or $F = \pi^2$ or $F \propto r^2$ *or* $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2} = \frac{1}{4}$

9 **(d)**

$A = (0.1)^2 = 0.01m^2$,

$\eta = 0.01\ Poise = 0.001\ decapoise$ (M.K.S. unit),

$dv = 0.1\ m/s$ and $F = 0.002\ N$

$F = \eta A \frac{dv}{dx}$

$\therefore dx = \frac{\eta A dv}{F} = \frac{0.001 \times 0.01 \times 0.1}{0.002} = 0.0005m$

10 **(d)**

$\alpha = \frac{\beta}{2} = \frac{2 \times 10^{-5}}{2} = 10^{-5}/°\text{C}$

11 **(a)**

Efficiency of engine $\eta = \frac{\text{Work done}}{\text{Heat in put}}$

Also, $\eta = 1 - \frac{T_2}{T_1}$

$$\frac{W}{Q} = 1 - \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{12.6 \times 10^6}{Q} = 1 - \frac{27 + 273}{927 + 273}$$

$$Q = 16.8 \times 10^6 \text{J}$$

12 **(c)**

$n_1 C_{v1} \Delta T_1 = n_2 C_{v2} \Delta T_2$

$\Rightarrow n_1 \times \frac{3}{2}R \times 10 = n_2 \times \frac{5}{2}R \times 6 \Rightarrow \frac{n_1}{n_2} = 1$

13 **(c)**

Here, $y_1 = 5(\sin 2\pi t + \sqrt{3}\cos 2\pi t)$

$y_2 = 5\sin\left(2\pi t + \frac{\pi}{4}\right)$

$y_1 = 5\sin 2\pi t + 5\sqrt{3}\cos 2\pi t$

As of the form of $y_1 = \alpha \sin 2\pi t + \beta \cos 2\pi t$

Let $\alpha = r\cos\theta = 5, \beta = r\sin\theta = 5\sqrt{3}$

$\therefore y_1 = r\cos\theta \sin 2\pi t + r\sin\theta \cos 2\pi t$

$= r\sin(2\pi t + \theta)$

Also, $\alpha^2 + \beta^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2$

$\Rightarrow r = \sqrt{\alpha^2 + \beta^2} = \sqrt{(5)^2 + \left(5\sqrt{3}\right)^2}$

$= 5\sqrt{1^2 + \left(\sqrt{3}\right)^2} = 10$

$\therefore y_1 = 10\sin(2\pi t + \theta)$

$\therefore \frac{A_1}{A_2} = \frac{10}{5} = \frac{2}{1}$

14 **(a)**

Here ,$T_1 = 16\ N, T_2 =$?

As per the choice given, $T_2 > T_1$

$\therefore n_2 > n_1, (n_2 - n_1) = 3 \qquad \text{...}(i)$

As $n \propto \sqrt{T}$

$\therefore \frac{n_2}{n_1} = \sqrt{\frac{T}{16}} = \sqrt{\frac{T}{4}}$

If n_1 corresponds to 4: n_2 corresponds to $3 + 4 = 7$, which is $\sqrt{T}$. Therefore, $T = 49$ N

16 **(d)**

$W = Q\,dV = Q(V_q - V_p)$

$= -100 \times (1.6 \times 10^{-19}) \times (-4 - 10)$

$= +100 \times 1.6 \times 10^{-19} \times 14$

$= +2.24 \times 10^{-16} J$

17 **(c)**

Current drawn from the cell in resistance R_1 will be $I = E/(R_1 + r)$

Therefore, heat produced in R_1 ie,

$H_1 = \dfrac{E^2 R_1 t}{(R_1 + r)^2}$

Heat produced in R_2 $ie, H_2 = \frac{E^2 R_2 t}{(R_2 + r)^2}$

As per question $H_1 = H_2$

or $\frac{E^2 R_1 t}{(R_1 + r)^2} = \frac{E^2 R_2 t}{(R_2 + r)^2}$

On solving we get;

$r = \sqrt{R_1 R_2}$

$= \sqrt{100 \times 40}$=63.25Ω

18 **(d)**

Let $m_1 = m$, then $m_2 = 3m$ and $m_3 = 5m$. Again let $L_3 = l$, then $L_2 = 3l$ and $L_1 = 5l$. If σ be the density of copper, then

$A_1 = \dfrac{m_1}{L_1 \sigma} = \dfrac{m}{5l\sigma}, A_2 = \dfrac{3m}{3l\sigma}$ and $A_3 = \dfrac{5m}{l\sigma}$

Hence $A_2 = \frac{5A_1}{3}$ and $A_3 = 25A_1$

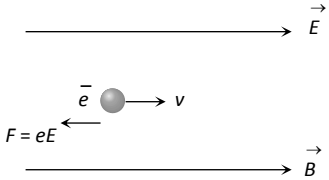
$\therefore\ R_1 = \dfrac{\rho L_1}{A_1} = \dfrac{\rho.5l}{A_1}, R_2 = \dfrac{\rho L_2}{A_2} = \dfrac{\rho.3l}{5A_1} = \dfrac{3}{25}R_1,$

and $R_3 = \frac{\rho L_3}{A_3} = \frac{\rho.l}{25A_1} = \frac{R_1}{125}$

$\therefore\ R_1 : R_2 : R_3 = R_1 : \dfrac{3}{25}R_1 : \dfrac{R_1}{125} = 125 : 15 : 1$

19 **(d)**

Since electron is moving parallel to the magnetic field, hence magnetic force on it $F_m = 0$



The only force acting on the electron is electric force which reduces it's speed

20 **(c)**

In equilibrium, the resultant magnetic moment will be along magnetic meridian. Let $N_1 S_1$ make $\angle\theta$ with resultant

$\tan\theta = \dfrac{M_2}{M_1} = \dfrac{M}{\sqrt{3}M} = \dfrac{1}{\sqrt{3}}\ \therefore\ \theta = 30°$

22 **(b)**

For purely capacitive circuit $e = e_0 \sin\omega t$

$i = i_0 \sin\left(\omega t + \dfrac{\pi}{2}\right)$, *i. e.*, current is ahead of emf by $\frac{\pi}{2}$

24 **(b)**

Power of the system decreases due to separation between the lenses. So, the focal length increases

26 **(b)**

For emission of electrons incident energy of each photon must be greater than work function (threshold energy)

27 **(b)**

The minimum energy needed to ionise an atom is called ionisation energy. The potential difference through which an electron should be accelerated to acquire this much energy is called ionisation potential.

$(E_2)_H - (E_1)_H = 10.2$ eV

or $\frac{(E_1)_H}{4} - (E_1)_H = 10.2$ eV

$\therefore\ (E_1)_H = -13.6$ eV

Hence ,ionisation potential energy is

$= (E_\infty)_H - (E_1)_H = 13.6$ eV

$\therefore$ Ionisation potential = 13.6 V

28 **(a)**

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ *or* $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/t_{1/2}}$ or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/1}$

For $t = 3$ months

$\frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

Therefore , disintegrated part of substance in 3 months

$= 1 - \frac{1}{8} = \frac{7}{8}$

29 **(b)**

Voltage amplification (gain) is given by

$A = \dfrac{\mu}{1 + \frac{r_p}{R_L}}$

where μ is amplification factor, r_P is plate resistance and R_L is load resistance.

Given, $\mu = 36, r_P = 10000\ \Omega, R_L = 30000\ \Omega$

Putting in the relation, we obtain

$A = \dfrac{36}{1 + \frac{10000}{30000}} = \dfrac{36}{1 + \frac{1}{3}} = \dfrac{36 \times 3}{4} = 27$

30 **(b)**

Characteristic impedance of a coaxial cable is

between 50 Ω to 70 Ω.

31 **(c)**

$Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$

1 mol 3 mol

Volume of 1 mole carbon monoxide

=22.4 L (at STP)

1 mole of ferric oxide is reduced by=3 moles of CO

$= 3 \times 22.4$ L of CO

$= 67.2$ dm^3 of CO

32 **(a)**

$\lambda = \frac{h}{mv}$

$= \frac{6.63 \times 10^{-34}}{60 \times 10^{-3} \times 10}$

$= 1.105 \times 10^{-33}$m

33 **(b)**

In the Periodic Table metals usually used as catalysts belong to *d*-block *e.g.*, Ni, Pt etc.

34 **(a)**

O_2^{2-} (Total number of electrons =18)

$\sigma 1s^2, {}^*_\sigma 1s^2, \sigma 2s^2, {}^*_\sigma 2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2 {}^*_\pi 2p_x^2$

$= {}^*_\pi 2p_y^2$

35 **(c)**

CH_3OCH_3 lacks H-bonding hence, it is most volatile, so it has maximum vapour pressure

36 **(b)**

$XY \rightarrow X(g) + Y(g);\ \Delta H = +a$ kJ/mol ...(i)

$X_2 \rightarrow 2X;\ \Delta H = +a$ kJ/mol ...(ii)

$Y_2 \rightarrow 2Y;\ \Delta H = +0.5a$ kJ/mol ...(iii)

$\frac{1}{2} \times$ (ii) $+ \frac{1}{2} \times$ (iii) $-$ (i) gives

$\frac{1}{2}X_2 + \frac{1}{2}Y_2 \rightarrow XY$

$\Delta H = \left(+\frac{a}{2} + \frac{0.5}{2}a - a\right)$ kJ/mol

$\therefore\ -200 = +\frac{a}{2} + \frac{0.5a}{2} - a$

or $a = 800$

37 **(c)**

$K_p = \frac{p_{CO}^2}{p_{CO_2}} = \frac{4 \times 4}{2} = 8$

38 **(a)**

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$

39 **(d)**

Water is oxidised to oxygen by fluorine as

$$2F_2 + 2H_2O \rightarrow O_2 + 4HF$$

40 **(a)**

Sodium metal is manufactured by the electrolysis of fused sodium chloride mixed with KCl and KF.

On electrolysis ;

At iron cathode

$$Na^+ + e^- \longrightarrow Na\ (s)(\text{metallic sodium})$$

At graphite anode :

$$2Cl^- \longrightarrow Cl_2(g) + 2e^-$$

NaCl melts at 800°C. It is difficult to attain and maintain its melting point. So, KCl and KF are mixed to lower the melting point of NaCl to about 600°C. KCl and KF are themselves not electrolysed under the voltage conditions used for sodium.

43 **(a)**

$Cl_3 - C - CH = CH_2 \xrightarrow[\text{anti-Markownikoff rule}]{\text{HCl}} Cl_3 - C - CH_2 - CH_2Cl$

($\because$ $CCl_3$is highly electron attracting group)

45 **(b)**

Packing fraction of ccp $= \frac{\pi}{3\sqrt{2}} = 0.74 \Rightarrow 74\%$

% free space in ccp $= 26\%$

Packing fraction of bcc $= \frac{\pi\sqrt{3}}{8} = 0.68 \Rightarrow 68\%$

% free space in bcc $= 32\%$

46 **(d)**

We have, i=1-$\frac{x}{2}$

where, x = degree of association

Here, i=0.54

0.54=1-$\frac{x}{2}$

or $0.54 - 1 = -\frac{x}{2}$

or $-0.46 = -\frac{x}{2}$

x= 0.92

47 **(b)**

$Cr_2O_7^{2-} + 6e^- \longrightarrow 2Cr^{3+}$

Reduction of 1 mol of $Cr_2O_7^{2-}$ to Cr^{3+} required 6 moles of electrons. Hence, charge required = 2 × 96500 C

48 **(c)**

$k = Ae^{-\frac{E_a}{RT}}$

k =rate constant

A =pre-exponential, frequency factor

E_a =activation energy

R =gas constant

T =temperature

49 **(d)**

Colloidal solution of $CuCl_2$ is not prepared by double decomposition method

50 **(a)**

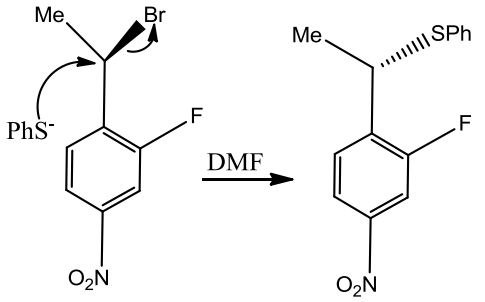
$$Ti + 2I_2 \xrightarrow{500\ K} \underset{\text{volatile stable compound}}{TiI_4} + 2I_2 \xrightarrow{1700\ K} \underset{\text{pure metal}}{Ti}$$

51 **(b)**

Welding of Mg is done in the atmosphere of He due to its inert and non-inflammable nature

54 **(a)**

PhS^- is a strong nucleophile and dimethyl formamide (DMF) is a highly polar aprotic solvent. Condition indicates that nucleophilic substitution (S_N2) takes place at 2° benzylic place, stereochemically, it involves inversion of configuration.



55 **(b)**

Because the difference in mass between $-CH_2OH$ group and $-COOH$ group is 14, thus the compound which undergoes oxidation is a primary alcohol. ($-CH_2OH$ is the functional group of primary alcohols).

$$\underset{(R+31)}{R\underset{|}{C}H_2OH} \rightarrow \underset{(R+45)}{RCOOH}$$

Primary alcohol acid

56 **(b)**

Aldehydes, which have no α −hydrogen atom, undergo Cannizaro reaction is presence of conc. NaOH and yield an alcohol and an acid salt. (Disproportionation).

$$\underset{\text{benzaldehyde}}{2C_6H_5CHO} \xrightarrow{NaOH} \underset{\text{benzyl alcohol}}{C_6H_5CH_2OH} + C_6H_5COONa$$

57 **(a)**

$$CH_3CH_2I \xrightarrow{NaCN} CH_3CH_2CN \xrightarrow{OH^-} CH_3CH_2CONH_2 \xrightarrow{Br_2/NaOH} CH_3CH_2NH_2$$

59 **(a)**

CH_3
|

(a) $H_2C = C - CH = CH_2$ is isoprene of 2-methyl 1, 3-butadiene. It is a monomer of natural rubber.

CH_3
|

(b) $H_2C = C - CH = CH_2$ is chloroprene or 2-chloro 1, 3-butadiene. It is a monomer of neoprene.

(c) $C_6H_5CH = CH_2$ is styrene. It is copolymer of buna-S rubber.

(d)

$$\left(-CH_2-CH_2-\right)_n$$

61 **(d)**

$$\because \quad n(A \times B \times C) = n(A) \times n(B) \times n(C)$$

$$\therefore \quad n(C) = \frac{24}{4 \times 3} = 2$$

62 **(a)**

Here, $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$, to find domain we must have,

$$\sin^{-1}(2x) + \frac{\pi}{6} \geq 0$$

$$\left(\text{but } -\frac{\pi}{2} \leq \sin^{-1}\theta \leq \frac{\pi}{2}\right)$$

$$\therefore \quad -\frac{\pi}{6} \leq \sin^{-1}(2x) \leq \frac{\pi}{2}$$

$$\Rightarrow \sin\left(-\frac{\pi}{6}\right) \leq 2x \leq \sin\left(\frac{\pi}{2}\right)$$

$$\Rightarrow -\frac{1}{2} \leq 2x \leq 1 \Rightarrow x \in \left[-\frac{1}{4}, \frac{1}{2}\right]$$

63 **(d)**

$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$

$$= \frac{1}{\sqrt{10}} \cdot \sqrt{1 - \frac{1}{5}} + \frac{1}{\sqrt{5}} \sqrt{1 - \frac{1}{10}}$$

$$\left[\because \sin A = \frac{1}{\sqrt{10}}, \sin B = \frac{1}{\sqrt{5}}\right]$$

$$= \frac{1}{\sqrt{10}} \sqrt{\frac{4}{5}} + \frac{1}{\sqrt{5}} \sqrt{\frac{9}{10}} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

$$\Rightarrow \quad A + B = \frac{\pi}{4}$$

64 **(a)**

As we have $A^2 = 2A - I$

$$\Rightarrow \quad A^2 A = (2A - I)A = 2A^2 - IA$$

$$\Rightarrow \quad A^3 = 2(2A - I) - IA = 3A - 2I$$

Similarly, $A^4 = 4A - 3I$

$A^5 = 5A - AI$

$A^n = nA - (n-1)I$

65 **(d)**

Let $\sqrt{6+4\sqrt{3}} = \sqrt{x} + \sqrt{y}$

$\Rightarrow 6 + 4\sqrt{3} = x + y + 2\sqrt{xy}$

$\Rightarrow \ x + y = 6, \ \sqrt{xy} = 2\sqrt{3}$

Now, $(x-y)^2 = (x+y)^2 - 4xy$

$= 36 - 4(4 \times 3)$

$= -12 < 0$

It is not possible

Hence, square root is not possible

66 **(d)**

$x^{12} - x^9 + x^4 - x + 1 > 0$, three cases arise

Case I When $x \leq 0$

$x^{12} > 0, -x^9 > 0, x^4 > 0, -x > 0$

$\Rightarrow \ x^{12} - x^9 + x^4 + x + 1 > 0, \forall\, x \leq 0$ ...(i)

Case II When $0 < x \leq 1$

$x^9 < x^4, x < 1 \ \Rightarrow \ -x^9 + x^4 > 0$ and $1 - x > 0$

$\therefore \ x^{12} - x^9 + x^4 - x + 1 > 0, \forall\, 0 < x \leq 1$ ...(ii)

Case III When $x > 1$

$x^{12} > x^9, x^4 > x$

$\Rightarrow \ x^{12} - x^9 + x^4 - x + 1 > 0, \forall\, x > 1$ ...(iii)

$\therefore$ From Eqs. (i), (ii) and (iii) the above equation hold for $x \in R$

67 **(d)**

(1)It is true that product of r consecutive natural numbers is always divisible by r.

(2) Now, $115500 = 2^2 \times 3^1 \times 5^3 \times 7^1 \times 11^1$

$\therefore$ Total number of proper divisor

$= (2+1)(1+1)(3+1)(1+1)(1+1) - 2$

$= 96 - 2 = 94$

(3) Total number of ways $= \frac{52!}{(13!)^4}$

Hence, all statements are true

68 **(d)**

Let $P(n) = n^3 + 2n$

$\Longrightarrow P(1) = 1 + 2 = 3$

$\Longrightarrow P(2) = 8 + 4 = 12$

$\Longrightarrow P(3) = 27 + 6 = 33$

Here, we see that all these number are divisible by 3

69 **(c)**

$\left(a + \frac{1}{p} - \frac{1}{q}\right)^2 + \left(a + \frac{1}{q} - \frac{1}{r}\right)^2 + \left(a + \frac{1}{r} - \frac{1}{s}\right)^2 \leq 0$

$\Rightarrow \frac{1}{p} - \frac{1}{q} = \frac{1}{q} - \frac{1}{r} = \frac{1}{r} - \frac{1}{s}$

$\Rightarrow \ p, q, r, s$ are in HP.

70 **(a)**

Given lines are concurrent, then

$$\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$$

$\Rightarrow 1(-4b-2) - 3(-16+4) - 9(-4-2b) = 0$

$\Rightarrow 14b + 70 = 0 \ \Rightarrow \ b = -5$

71 **(b)**

We know that, if (x_1, y_1) is the mid point of the chord, then equation of chord is

$T = S_1 \Rightarrow \frac{xx_1}{25} + \frac{yy_1}{9} = \frac{x_1^2}{25} + \frac{y_1^2}{9}$

$\because$ Point is $(1, 1)$, then

$\frac{x}{25} + \frac{y}{9} = \frac{1}{25} + \frac{1}{9}$

$\Rightarrow 9x + 25y = 34$

72 **(c)**

$$\lim_{x\to\infty}\left(\frac{x+3}{x+1}\right)^{x+2} = \lim_{x\to\infty}\left[\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)}\right]^{\frac{2}{x+2}\times(x+2)}$$

$$= \lim_{x\to\infty}\left(1+\frac{2}{x+1}\right)^{\left(\frac{x+2}{2}\right)\left[\frac{2(x+2)}{x+1}\right]}$$

$$= e^{\lim_{x\to\infty}\frac{(2+4/x)}{(1+1/x)}}$$

$$= e^2$$

73 **(b)**

$\sim (p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q)$

$\equiv \sim p$

74 **(a)**

$\because$ Mode=3 Median-2 Mean

$\because$ Mode=3(22)-2(21)

$\Rightarrow$ Mode=66-42=24

75 **(d)**

Let $E = E$ =Events of numbers divisible by 2 and 3 [ie, divisible by 6]

$= (6, 12, \ldots, 96)$

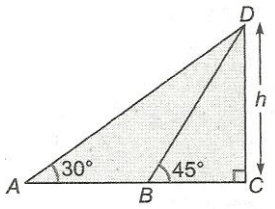
$n(E) = 16$

$\therefore$Required probability $= \frac{{}^{16}C_3}{{}^{100}C_3}$

$$= \frac{\frac{16\times15\times14}{3\times2\times1}}{\frac{100\times99\times98}{3\times2\times1}} = \frac{4}{1155}$$

76 **(c)**

Let DC be the height of building



$\therefore AB = \text{Speed} \times \text{Time}$

$= 25(\sqrt{3}-1).2$

$= 50(\sqrt{3}-1)$

$\therefore$ In $\Delta DBC, \tan 45° = \dfrac{DC}{BC} \Rightarrow BC = h$

In $\Delta DAC, \tan 30° = \dfrac{h}{50(\sqrt{3}-1)+h}$

$\Rightarrow 50(\sqrt{3}-1) + h = \sqrt{3}h$

$\Rightarrow h = 50$ m

77 **(b)**

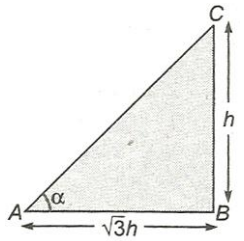
Let the height of the tower be $BC = h$,then length of shadow of tower $AB = \sqrt{3}h$.

In $\Delta ABC, \tan\alpha = \frac{BC}{AB}$

$\Rightarrow \tan\alpha = \dfrac{h}{\sqrt{3h}}$

$\Rightarrow \tan\alpha = \tan 30°$

$\Rightarrow \alpha = 30°$



78 **(a)**

$f(x) = \log_5(\log_7 x)$

$\Rightarrow f(x) = \log_5\left(\dfrac{\log_e x}{\log_e 7}\right)$

$\Rightarrow f(x) = \log_5 \log_e x - \log_5 \log_e 7$

$\Rightarrow f(x) = \dfrac{\log_e \log_e x}{\log_e 5} - \log_5 \log_e 7$

On differentiating w.r.t. x, we get

$f'(x) = \dfrac{1}{x \log_e x \log_{e\,5}} - 0$

$= \dfrac{1}{\frac{x \log_e x}{\log_e 7} \log_e 7.\log_e 5}$

$= \dfrac{1}{x \log_7 x \log 7.\log 5}$

79 **(a)**

Given, $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$

$\therefore \quad \dfrac{\pi}{2} - \cos^{-1} x + \dfrac{\pi}{2} - \cos^{-1} y = \dfrac{\pi}{2}$

$\Rightarrow \cos^{-1} x + \cos^{-1} y = \dfrac{\pi}{2}$

80 **(a)**

The given system of equation has at least one solution, if

$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{vmatrix} = 0$

$\Rightarrow 3(2\lambda + 15) + 1(\lambda + 18) + 4(5 - 12) = 0$

$\Rightarrow 7\lambda = -35 \Rightarrow \lambda = -5$

81 **(b)**

Given, $f(x) = \begin{vmatrix} x & 1+\sin x & \cos x \\ 1 & \log(1+x) & 2 \\ x^2 & 1+x^2 & 0 \end{vmatrix}$

$= x\{-2(1+x^2)\} - (1+\sin x)(-2x^2)$

$+ \cos x\{1 + x^2 - x^2 \log(1+x)\}$

$= -2x - 2x^3 + 2x^2 + 2x^2 \sin x$

$+ \cos x\{1 + x^2 - x^2 \log(1+x)\}$

$\therefore$ Coefficient of x in $f(x) = -2$

82 **(b)**

For $x > 1$, we have

$f(x) = |\log|x|| = \log x \quad \Rightarrow \quad f'(x) = \dfrac{1}{x}$

For $x < -1$, we have

$f(x) = |\log|x|| = \log(-x) \quad \Rightarrow \quad f'(x) = \dfrac{1}{x}$

For $0 < x < 1$, we have

$f(x) = |\log|x|| = -\log x \quad \Rightarrow \quad f'(x) = \dfrac{-1}{x}$

For $-1 < x < 0$, we have

$f(x) = -\log(-x) \quad \Rightarrow \quad f'(x) = -\dfrac{1}{x}$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

83 **(c)**

Given , $t = as^2 + bs + c$

$\Rightarrow 1 = 2as\frac{ds}{dt} + b\frac{ds}{dt}$ [differentiating]

$\Rightarrow 1 = 2asv + bv$ ...(i)

$\Rightarrow 0 = 2a\frac{ds}{dt}v + 2as\frac{dv}{dt} + b\frac{dv}{dt}$ [differentiating]

$\Rightarrow \dfrac{dv}{dt}(2as + b) = -2av^2$

$\Rightarrow \frac{dv}{dt}\left(\frac{1}{v}\right) = -2av^2$ [from Eq.(i)]

$\Rightarrow \dfrac{dv}{dt} = -2av^3$

85 **(d)**

Let $I = \int_0^{\pi/2} \frac{dx}{1+\cot x} = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\, dx$ ...(i)

$\Rightarrow I = \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{\sin x}{\cos x+\sin x}\, dx$ ...(ii)

On adding Eqs.(i) and (ii), we get

$2I = \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$

$\Rightarrow\ I = \frac{\pi}{4}$

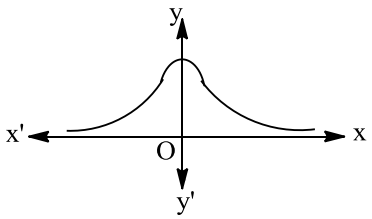
86 **(c)**

The figure of the given curve $y = \frac{a^3}{a^2+x^2}$ is

$\therefore$ Required volume

$V = 2\int_0^{\infty} \pi y^2\, dx$

$= 2\pi a^6 \int_0^{\infty} \frac{1}{(a^2+x^2)^2}\, dx$



Put $x = a\tan\theta$

$\Rightarrow\ dx = a\sec^2\theta\, d\theta$

$\therefore V = 2\pi a^6 \int_0^{\pi/2} \frac{a\sec^2\theta}{(a^2+a^2\tan^2\theta)^2} d\theta$

$= \frac{2\pi a^6}{a^3}\int_0^{\pi/2} \cos^2\theta d\theta = 2\pi a^3\left[\frac{1}{2}\cdot\frac{\pi}{2}\right]$

$= \frac{\pi^2 a^3}{2}$ cu units

87 **(c)**

Given, $y^2 = 4ax + 4a^2$ ...(i)

$\Rightarrow\quad 2yy' = 4a$

On putting the value of $4a$ in eq(i), we get

$y^2 = 2yy'x + 4.\frac{y^2y'^2}{4}$

$\Rightarrow\quad y = 2y'x + yy'^2$

88 **(a)**

In plane P_1, a vector is perpendicular to $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ is $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$.

In plane P_2, a vector is perpendicular to $\vec{\mathbf{c}}$ and $\vec{\mathbf{d}}$ is $\vec{\mathbf{c}} \times \vec{\mathbf{d}}$

$\Rightarrow (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \times (\vec{\mathbf{c}} \times \vec{\mathbf{d}}) = 0$

$\Rightarrow\ (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) || (\vec{\mathbf{c}} \times \vec{\mathbf{d}})$

The angle between the planes is 0.

89 **(b)**

Let the equation of any plane passing through $P(2,-1,3)$ is

$a(x-2) + b(y+1) + c(z-3) = 0$ .....(i)

$\therefore$ DR's of $OP = 2,-1,3$

Since, the line OP is perpendicular to the plane, therefore the DR's of the normal to the plane is proportional to the DR's of OP.

$\therefore$ Required equation of plane is

$2(x-2) - 1(y+1) + 3(z-3) = 0$

$\Longrightarrow 2x - y + 3z - 14 = 0$

90 **(b)**

For maximum profit $z = 40x + 25y$