THOMAStutorials

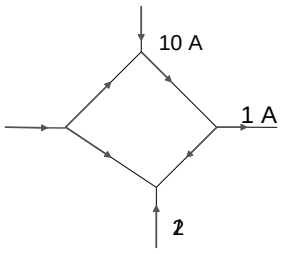
JEE (FINAL)

Date : **TEST NO:** 58

Time : 3 HRS **PCM** **MARK:** 360

Single Correct Answer Type

1. A particle A has charge $+q$ and a particle B has charge $+4q$ with each of them having the same mass m. When allowed to fall from rest through the same electric potential difference, the ratio of their speed $\frac{v_A}{v_B}$ will become
   a) 2:1 b) 1:2 c) 1:4 d) 4:1
2. $ABCD$ is a rectangle. At corners B, C and D of the rectangle are placed charges $+10 \times 10^{-10}$C, -20×10^{-12}C, and 10×10^{-12}C, respectively. Calculate the potential at the fourth corner. (The side AB=4cm and BC=3cm)
   a) 1.65 V b) 0.165V c) 16.5V d) 2.65V
3. A bulb of 220 V and 300 W is connected across 110 V circuit. The percentage reduction in power is
   a) 100% b) 25% c) 70% d) 75%
4. The figure shows a network of currents. The magnitude of current is shown here. The current I will be



   a) 3A b) 9A c) 13A d) 19A
5. When a magnetic field is applied in a direction perpendicular to the direction of cathode rays, then their
   a) Energy decreases
   b) Energy increases
   c) Momentum increases
   d) Momentum and energy remain unchanged
6. A magnetic dipole is placed at right angles to the direction of lines of force of magnetic induction B. If it is rotated through an angle of 180°, then the work done is
   a) MB b) $2\,MB$ c) $-2\,MB$ d) Zero
7. According to Lenz's law of electromagnetic induction
   a) The induced emf is not in the direction opposing the change in magnetic flux.
   b) The relative motion between the coil and magnet produces change in magnetic flux
   c) Only the magnet should be moved towards coil
   d) Only the coil should be moved towards magnet
8. The primary winding of a transformer has 200 turns and its secondary winding has 50 turns. If the current in the secondary winding is 40 A, the current in the primary is
   a) 10 A
   b) 80 A
   c) 160 A
   d) 800 A
9. A radiowave has a maximum magnetic field induction of 10^{-4} T on arrival at a receiving antenna. The maximum electric field intensity of such a wave is
   a) Zero b) 3×10^4 Vm^{-1}
   c) 5.8×10^{-4} T d) 3.0×10^{-5} T
10. A convex lens is placed between object and a screen. The size of object is 3 cm and an image of height 9 cm is obtained on the screen. When the lens is displaced to a new position, what will be the size of image on the screen?
    a) 2 cm b) 6 cm c) 4 cm d) 1 cm
11. In young's two slit experiment the distance between the two coherent sources is 2 mm and the screen is at a distance of 1 m. If the fringe width is found to be 0.03 cm, then the wavelength of the light used is
    a) 4000Å b) 5000Å
    c) 5890Å d) 6000Å
12. A charged particle is moving in the presence of electric field $\vec{\mathbf{E}}$ and magnetic field $\vec{\mathbf{B}}$. The directions of $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ are such that the charged particle moves in a straight line and its speed

increases. The relations amongst $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$ and velocity $\vec{\mathbf{v}}$ must be such that

a) $\vec{\mathbf{E}} \cdot \vec{\mathbf{B}} = 0, \vec{\mathbf{v}}$ is arbitrary

b) $\vec{\mathbf{E}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{v}}$ are all parallel to each other

c) $\vec{\mathbf{E}} \cdot \vec{\mathbf{v}} = 0; \vec{\mathbf{B}} \cdot \vec{\mathbf{v}} = 0$ but $\vec{\mathbf{E}} \cdot \vec{\mathbf{B}} \neq 0$

d) $\vec{\mathbf{v}}$ is parallel to $\vec{\mathbf{E}}$ and perpendicular to $\vec{\mathbf{B}}$

13. The energy of an electron in an excited hydrogen atom is -3.4 eV. Its angular momentum is

a) 3.72×10^{-34} Js b) 2.11×10^{-34} Js

c) 1.57×10^{-34} Js d) 1.11×10^{-34} Js

14. Consider an electron in the n^{th} orbit of a hydrogen atom in the Bohr model. The circumference of the orbit can be expressed in terms of the de Broglie wavelength λ of that electron as

a) $(0.259)n\lambda$ b) $\sqrt{n}\lambda$

c) $(13.6)\lambda$ d) $n\lambda$

15. A logic gate is an electronic circuit which

a) Makes logic decisions

b) Allows electrons flow only in one direction

c) Works binary algebra

d) Alternates between 0 and 1 values

16. Modulation is the process of superposing

a) Low frequency audio signal on high frequency waves

b) Low frequency radio signal on low frequency audio waves

c) High frequency audio signal on low frequency radio waves

d) Low frequency audio signal on low frequency radio waves

17. How many number of atoms are there in a cube based unit cell having one atom on each corner and two atoms on each body diagonal of cube

a) 8 b) 6 c) 4 d) 9

18. Which of the following can be measured by the Ostwald-Walker dynamic method?

a) Relative lowering of vapour pressure

b) Lowering of vapour pressure

c) Vapour pressure of the solvent

d) All of the above

19. The standard emf of a cell involving one electron change is found to be 0.591 V and 25 °C. The equilibrium constant of the reaction is ($F = 96500$ C mol^{-1})

a) 1.0×10^{1} b) 1.0×10^{5}

c) 1.0×10^{10} d) 1.0×10^{30}

20. If the half-time for a particular reaction is found to be constant and independent of the initial concentration of the reactants, then the ratio is of

a) First order b) Zero order

c) Second order d) None of these

21. A biological catalyst is

a) An amino acid

b) A carbohydrate

c) The nitrogen molecule

d) An enzyme

22. Of the following, which cannot be obtained by electrolysis of the aqueous solution of their salts?

a) Cu b) Ag c) Mg and Al d) Cr

23. ClO^- disproportionate into

a) Cl^- and O b) Cl^- and ClO_3^-

c) Cl and O d) $Cl^-$and O^-

24. The lanthanide contraction is responsible for the fact that

a) Zr and Zn have the same oxidation state

b) Zr and Hf have about the same radius

c) Zr and Nb have similar oxidation state

d) Zr and Y have about the same radius

25. The trivial name among the following is

a) Acetone b) Acetylene

c) Uric acid d) None of these

26. The product of vinyl chloride and HCl is a

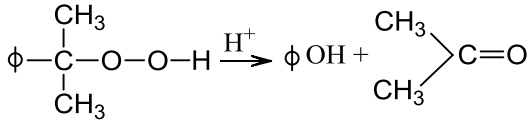
a) *gem* chloride

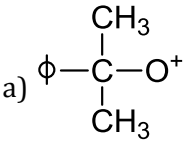
b) Ethylidene chloride

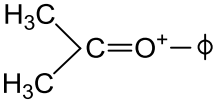
c) 1, 1 dichloroethane

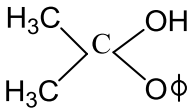
d) All of the above are correct

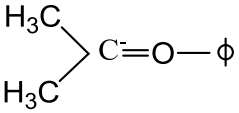
27. Which doesn't form in the acid catalysed rearrangement of cumene hydroperoxide?



a) 

b) 

c) 

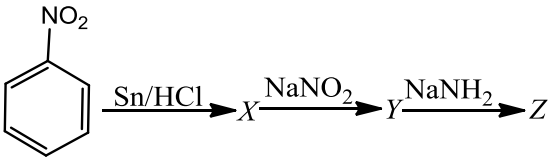
d) 

28. 3-hydroxybutanal is formed when (*X*) reacts with (*Y*)in dilute (*Z*) solution. What are *X,* Y and Z?

	X	*Y*	*Z*
a)	CH_3CHO,	$(CH_3)_2CO$,	NaOH

b) CH_3CHO, CH_3CHO, NaCl
c) $(CH_3)_2CO$, $(CH_3)_2CO$, HCl
d) CH_3CHO, CH_3CHO, NaOH

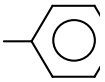
29. What is 'Z'in the following reaction ?



a) Benzoic acid b) Cyanobenzoic acid
c) Benzamide d) Aniline

30. Amino acids have peptide linkage which is
a) —CO—NH— b) —C—NH_2
c) SO—NH— d) —CO—N—

31. PVC is prepared by the polymerization of
a) Ethylene b) 1-chloropropene
c) Propene d) 1-chloroethene

32. Which detergent can cause maximum pollution?
a) $CH_3CH(C_4H_9)-CH_2CH_2CH(C_4H_9)CH(C_4H_9)CH_2CH_2SO_3$
b) 
c) 
d) Detergents are always pollution free

33. Derivative of $x^6 + 6^x$ with respect to x is
a) $12x$ b) $x + 4$
c) $6x^5 + 6^x \log 6$ d) $6x^5 + x6^{x-1}$

34. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is
a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$
c) $-\frac{1}{2}$ d) None of these

35. If the three linear equations
$x + 4ay + az = 0$
$x + 3by + bz = 0$
$x + 2cy + cz = 0$
Have a non-trivial solution, where
$a \neq 0, b \neq 0, c \neq 0$, then $ab + bc$ is equal to
a) $2ac$ b) $-ac$ c) ac d) $-2ac$

36. If $f(x)$, g(x) and $h(x)$ are three polynomials of degree 2 and $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ f'(x) & g'(x) & h'(x) \\ f''(x) & g''(x) & h''(x) \end{vmatrix}$, then $\Delta(x)$ is polynomial of degree
a) 2 b) 3
c) At most 2 d) At most 3

37. Function $f(x) = \begin{cases} x-1, x < 2 \\ 2x-3, x \geq 2 \end{cases}$ is a continuous function
a) For $x = 2$ only
b) For all real values of x such that $x \neq 2$
c) For all real values of x
d) For all integer values of x only

38. The values of a in order that $f(x) = \sqrt{3}\sin x - \cos x - 2ax + b$ decreases for all real values of x, is given by
a) $a < 1$ b) $a \geq 1$ c) $a \leq \sqrt{2}$ d) $a < \sqrt{2}$

39. If $I = \int \frac{x^5}{\sqrt{1+x^3}}\, dx$, then I is equal to
a) $\frac{2}{9}(1+x^3)^{\frac{5}{2}} + \frac{2}{3}(1+x^3)^{\frac{3}{2}} + c$
b) $\log\left|\sqrt{x} + \sqrt{1+x^3}\right| + c$
c) $\log\left|\sqrt{x} - \sqrt{1-x^3}\right| + c$
d) $\frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}} + c$

40. The value of the integral $\int \frac{dx}{x(1+\log x)^2}$ is equal to
a) $\frac{-1}{1+x} + c$ b) $\frac{-1}{1+\log x} + c$
c) $\frac{1}{1+\log x} + c$ d) $\frac{1}{1+x} + c$

41. The area bounded by the curves $y^2 = 4a^2(x-1)$and lines $x = 1$and $y = 4a$ is
a) $4a^2$ sq units b) $\frac{16a}{3}$ sq units
c) $\frac{16a^2}{3}$ sq units d) None of these

42. The solution of $\frac{dy}{dx} = 1 + y + y^2 + x + xy + xy^2$ is
a) $\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = x + x^2 + c$
b) $4\tan^{-1}\left(\frac{4y+1}{\sqrt{3}}\right) = \sqrt{3}(2x + x^2) + c$
c) $\sqrt{3}\tan^{-1}\left(\frac{3y+1}{3}\right) = 4\,(1 + x + x^2) + c$
d) $4\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \sqrt{3}(2x + x^2) + c$

43. If $\vec{\mathbf{a}}, \vec{\mathbf{b}}, \vec{\mathbf{c}}$ are three non-coplanar vectors then the vector equation $\vec{\mathbf{r}} = (1-p-q)\vec{\mathbf{a}} + p\vec{\mathbf{b}} + q\vec{\mathbf{c}}$ represent a
a) Straight line
b) Plane
c) Plane passing through the origin
d) Sphere

44. A line makes acute angles of α, β and γ with the coordinate axes such that
$\cos\alpha\cos\beta = \cos\beta\cos\gamma = \frac{2}{9}$
And $\cos\gamma\cos\alpha = \frac{4}{9}$,
Then $\cos\alpha + \cos\beta + \cos\gamma$ is equal
To

a) $\frac{25}{9}$ b) $\frac{5}{9}$ c) $\frac{5}{3}$ d) $\frac{2}{3}$

45. The region represented by the in equation system $x, y \geq 0, y \leq 6, x + y \leq 3$, is
    a) Unbounded in first quadrant
    b) Unbounded in first and second quadrants
    c) Bounded in first quadrant
    d) None of the above

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: ANSWER KEY :

1)	**b**	**2)**	**a**	**3)**	**d**	**4)**	**c**	**5)**	**d**	**6)**	**d**	**7)**	**b**
8)	**a**	**9)**	**b**	**10)**	**d**	**11)**	**d**	**12)**	**b**	**13)**	**b**	**14)**	**d**
15)	**a**	**16)**	**a**	**17)**	**d**	**18)**	**d**	**19)**	**c**	**20)**	**a**	**21)**	**d**
22)	**c**	**23)**	**b**	**24)**	**b**	**25)**	**c**	**26)**	**d**	**27)**	**d**	**28)**	**d**
29)	**d**	**30)**	**a**	**31)**	**d**	**32)**	**a**	**33)**	**c**	**34)**	**b**	**35)**	**a**
36)	**c**	**37)**	**c**	**38)**	**b**	**39)**	**d**	**40)**	**b**	**41)**	**b**	**42)**	**d**
43)	**b**	**44)**	**c**	**45)**	**c**								

: HINTS AND SOLUTIONS :

1 **(b)**

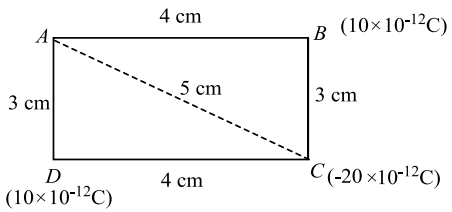
Using $v = \sqrt{\frac{2QV}{m}} \Rightarrow v \propto \sqrt{Q} \Rightarrow \frac{v_A}{v_B} = \sqrt{\frac{Q_A}{Q_B}} = \sqrt{\frac{q}{4q}} = \frac{1}{2}$

2 **(a)**

The situation is summarised in figure.

$BC = AD = 3\text{cm}, \quad AB = DC = 4\text{cm},$

So, $AC = 5\text{cm}.$



Now, potential at A

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_B}{AB} + \frac{1}{4\pi\varepsilon_0}.\frac{q_c}{AC} + \frac{1}{4\pi\varepsilon_0}.\frac{q_D}{AD}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{10\times10^{-12}}{4\times10^{-2}} - \frac{20\times10^{-12}}{5\times10^{-2}} + \frac{10\times10^{-12}}{3\times10^{-2}}\right]$$

$$= 9\times10^9\times10^{-10}\left[\frac{10}{4} - \frac{20}{5} + \frac{10}{3}\right]$$

$$= \frac{9\times10^{-1}\times11}{6}$$

$$= 16.5\times10^{-1} = 1.65\text{V}$$

3 **(d)**

Resistance of bulb,

$$R = \frac{V^2}{P} = \frac{220\times220}{300} = \frac{484}{3}\Omega$$

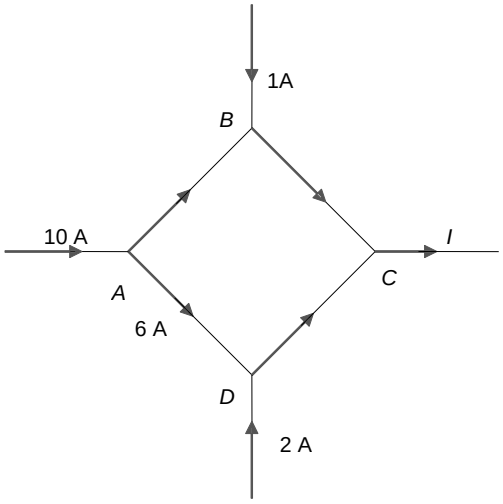
New power, $P' = \frac{(110)^2}{R} = \frac{110\times110}{484/3} = 75$ watt

reduction

Of power $= \frac{300-75}{300}\times100 = 75\%$

4 **(c)**

Regarding Kirchhoff's junction rule, the circuit can be redrawn as



Current in arm, $AB = 10 - 6 = 4\text{A}$

Current in arm, $DC = 6 + 2 = 8\text{A}$

Current in arm, $BC = 4 + 1 = 5\text{A}$

Hence, $I = 5 + 8 = 13\text{A}$

5 **(d)**
Since force is perpendicular to direction of motion, energy and magnitude of momentum remains constant

6 **(d)**
$\theta_1 = 90°, \theta_2 = 270°,$
$W = -MB[\cos 270° - \cos 90°]$ =zero

7 **(b)**
According to Lenz's law of electromagnetic induction, the relative motion between the coil and magnet produces change in magnetic flux.

8 **(a)**
$i_P = \frac{n_s}{n_P} i_s = \frac{50}{200} \times 40 = 10$ A

9 **(b)**
$E_0 = cB_0 = 3 \times 10^8 \times 10^{-4} = 3 \times 10^4$ Vm^{-1}

10 **(d)**
Form displacement method size of object,
$O = \sqrt{I_1 I_2}$
Here, $O = 3$ cm, $I_1 = 9$ cm
$\therefore 3 = \sqrt{9 I_2}$
Or $I_2 = 1$ cm

11 **(d)**
From $\beta = \frac{\lambda D}{d}$,

$$\lambda = \frac{\beta . d}{D} = \frac{0.3 \times 10^{-2} \times 2 \times 10^{-3}}{1} = 6 \times 10^{-7}\text{m}$$
$$= 6000 \text{ Å}$$

12 **(b)**
A charged particle moves along a straight line with acceleration, hence electric field should be parallel to the direction of motion of charged particle and no force should act on charged particle due to magnetic field. It will be so if charged particle is moving parallel to the direction of magnetic field

13 **(b)**
Energy of electron in nth energy level in hydrogen atom
$$= \frac{-13.6}{n^2} \text{ eV}$$
Here, $\frac{-13.6}{n^2} = -3.4$ eV
So, $n = 2$
Angular momentum from Bohr's principle
$$= \text{n}\frac{h}{2\pi} = \frac{2 \times 6.626 \times 10^{-34}}{2 \times 3.14}$$
$$= 2.11 \times 10^{-34} \text{ Js}$$

14 **(d)**
According to Bohr's theory $mvr = n\frac{h}{2\pi}$
$\Rightarrow$ Circumference $2\pi r = n\left(\frac{h}{mv}\right) = n\lambda$

16 **(a)**
Modulation is a process of superposing a low frequency audio signals (called modulating signal) on a high frequency radio wave called carrier wave

17 **(d)**
There are four body diagonals. Atoms on the body diagonals are not shared by any other unit cell.
Contribution by atoms on corners
$$= 8 \times \frac{1}{8} = 1 \text{ and}$$
Contribution by atoms on body diagonal
$$= 2 \times 4 = 8$$
Hence, total number of atoms = 9

18 **(d)**
By Ostwald-walker dynamic method, the relative lowering of vapour pressure, lowering of vapour pressure and vapour pressure of the solvent, all can be measured.
In this method, the apparatus used, contains two bulbs: bulb A contains solution and bulb B contains solvent. The loss of weight in bulb B gives the lowering vapour pressure and total loss of weight in both the tubes gives the vapour pressure of the solvent and
Relative lowering of vapour pressure

$$= \frac{\text{lowering of vapour pressure}}{\text{vapour pressure of solvent}}$$

19 **(c)**
$E^\circ_{\text{cell}} = \frac{2.303RT}{nF} \log K_{eq}$
$E^\circ_{\text{cell}} = \frac{0.0591}{n} \log K_{eq}$ [At 298 K]
$0.591 = \frac{0.0591}{1} \log K_{eq}$
$\therefore \log K_{eq} = 10$
$\therefore K_{eq} = 1 \times 10^{10}$

20 **(a)**
For first order reaction the half-life period is independent of the initial concentration of the reactants.
$t_{1/2} = \frac{0.693}{k}$

21 **(d)**
Enzymes are biological catalyst

22 **(c)**
Mg and Al cannot be obtained by the electrolysis of aqueous solution of their salts because instead of metal, H_2 gas is liberated at cathode

23 **(b)**

The hypochlorites disproportionate on heating as follows.

$3ClO^- \longrightarrow 2Cl^- + ClO_3^-$

24 **(b)**

Lanthanide contraction, cancels almost exactly the normal size increase on descending a group of transition elements, thus Nb and Ta, Zr and Hf have same covalent and ionic radii.

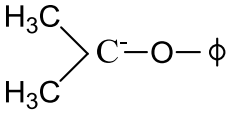
26 **(d)**

$CH_2 = CHCl + HCl \rightarrow CH_3 - CHCl_2$

ethylidene chloride
sor
1, 1 dichloroethane

27 **(d)**

In rearrangement of cumene hydroperoxide



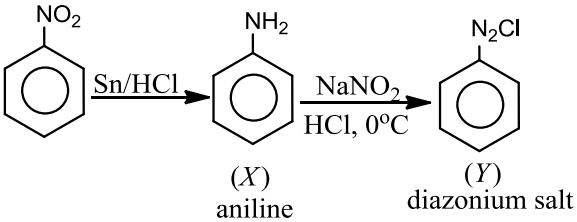
is not formed

28 **(d)**

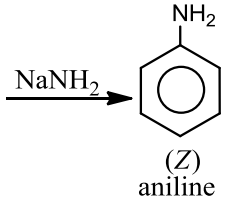
Aldehydes having α −H-atoms undergoes aldol condensation in the presence of dil.NaOH and yield β −hydroxy aldehydes.

$$CH_3CHO + CH_3CHO \xrightarrow{NaOH} \underset{\text{3-hydroxy butanal}}{CH_3\overset{OH}{\overset{|}{C}}H.\,CH_2CHO}$$

29 **(d)**



(X) aniline $\qquad$ (Y) diazonium salt



(Z) aniline

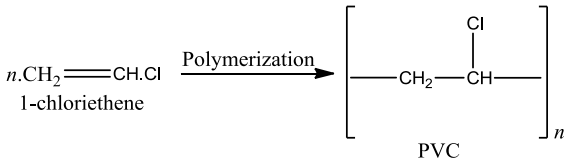
$\therefore Z$ is aniline

30 **(a)**

Two or more amino acids unite through a bond $(-CO - NH -)$ which is known as peptide bond or peptide linkage.

31 **(d)**

PVC is polyvinyl chloride, a polymer of vinyl chloride.



32 **(a)**

Greater the branching, greater is the non-biodegradability of a detergent. A non-biodegradable detergent cause maximum pollution

33 **(c)**

Let $y = x^6 + 6^x$

On differentiating w.r.t. x, we get

$\frac{dy}{dx} = 6x^5 + 6^x \log 6$

34 **(b)**

$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow 2\cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$

35 **(a)**

Given, $\quad x + 4ay + az = 0 \quad$ ...(i)

$x + 3by + bz = 0 \quad$ ...(ii)

And $\quad x + 2cy + cz = 0 \quad$ ...(iii)

For non-trivial solution

$\begin{vmatrix} 1 & 4a & a \\ 1 & 3b & b \\ 1 & 2c & c \end{vmatrix} = 0$

Applying $R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$\Rightarrow \begin{vmatrix} 1 & 4a & a \\ 0 & 3b - 4a & b - a \\ 0 & 2c - 4a & c - a \end{vmatrix} = 0$

$\Rightarrow 1[(3b - 4a)(c - a) - 2(b - a)(c - 2a)] = 0$

$\Rightarrow 3bc - 3ab - 4ac + 4a^2 - 2(bc - 2ab - ac + 2a^2) = 0$

$\Rightarrow \quad bc + ab - 2ac = 0$

$\Rightarrow \quad ab + bc = 2ac$

36 **(c)**

Let $f(x) = a_0x^2 + a_1x + a_2$

and $g(x) = b_2x^2 + b_1x + b_2$

Also, $h(x) = c_0x^2 + c_1x + c_2$

Then, $\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0x + a_1 & 2b_0x + b_1 & 2c_0x + c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$

$= x\begin{vmatrix} f(x) & g(x) & h(x) \\ 2a_0 & 2b_0 & 2c_0 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ 2a_0 & 2b_0 & 2c_0 \end{vmatrix}$

$= 0 + 2\begin{vmatrix} f(x) & g(x) & h(x) \\ a_1 & b_1 & c_1 \\ a_0 & b_0 & c_0 \end{vmatrix}$

$= 2[(b_1c_0 - b_0c_1)f(x) - (a_1c_0 - a_0c_1)g(x) + (a_1b_0 - a_0b_1)h(x)]$

Hence, degree of $\Delta(x) \leq 2$

37 **(c)**

Since, it is a polynomial function, so it is continuous for every value of x except at $x = 2$

LHL$= \lim_{x\to 2^-} x - 1$

$= \lim_{h\to 0} 2 - h - 1 = 1$

RHL$= \lim_{x\to 2^+} 2x - 3 = \lim_{h\to 0} 2(2 + h) - 3 = 1$

And $f(2) = 2(2) - 3 = 1$

$\therefore$ LHL+RHL$= f(2)$

Hence, $f(x)$ is continuous for all real values of x

38 **(b)**

Given, $f'(x) < 0, \forall\, x \in R$

$\Rightarrow \sqrt{3}\cos x + \sin x - 2a < 0, \forall\, x \in R$

$\Rightarrow \frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x < a, \forall\, x \in R$

$\Rightarrow \sin\left(x + \frac{\pi}{3}\right) < a, \forall\, x \in R$

$\Rightarrow a \geq 1 \left[\because \sin\left(x + \frac{\pi}{3}\right) \leq 1\right]$

39 **(d)**

Given, $I = \int \frac{x^5}{\sqrt{1+x^3}}dx$

Let $1 + x^3 = t \Rightarrow 3x^2dx = dt$

$\therefore I = \int \frac{(t-1)}{\sqrt{t}} \cdot \frac{dt}{3} = \frac{1}{3}\int \left(\sqrt{t} - t^{-1/2}\right) dt$

$= \frac{1}{3}\left[\frac{2t^{3/2}}{3} - 2t^{1/2}\right] + c$

$= \frac{2}{9}(1 + x^3)^{3/2} - \frac{2}{3}(1 + x^3)^{1/2} + c$

40 **(b)**

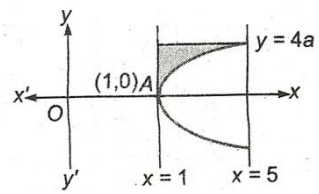
Put $1 + \log x = t \Rightarrow \frac{1}{x}dx = dt$

$\int \frac{dx}{x(1+\log x)^2} = \int t^{-2}\, dt = \frac{t^{-1}}{-1} + c$

$= \frac{-1}{(1+\log x)} + c$

41 **(b)**

On solving $y^2 = 4a^2(x - 1)$and $y = 4a$, we get $x = 5$



$\therefore$ Required area $= \int_1^5 \left(4a - 2a\sqrt{x-1}\right)dx$

$= \left[4ax - 2a\frac{(x-1)^{3/2}}{3/2}\right]_1^5$

$= \frac{16a}{3}$ sq units

42 **(d)**

Given, $\frac{dy}{1+y+y^2} = (1 + x)dx$

$\Rightarrow \int \frac{dy}{\left(y+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \int (1 + x)\, dx$

$\Rightarrow \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) = x + \frac{x^2}{2} + \frac{c}{2}$

$\Rightarrow 4\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) = \sqrt{3}(2x + x^2) + c$

43 **(b)**

Given equation represents a plane.

44 **(c)**

Given, $\cos\alpha\cos\beta\cos\gamma = \frac{2}{9}$

and $\cos\gamma\cos\alpha = \frac{4}{9}$

Then, $\cos\alpha = \frac{2}{3}, \cos\beta = \frac{1}{3}$ and $\cos\gamma = \frac{2}{3}$

$\therefore \cos\alpha + \cos\beta + \cos\gamma = \frac{2}{3} + \frac{1}{3} + \frac{2}{3} = \frac{5}{3}$

45 **(c)**

The given region is bounded in first quadrant