THOMAStutorials

JEE (FINAL)

Date : **TEST NO:** 60

Time : 3 HRS **PCM** **MARK:** 360

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1. The ratio of charge to potential of a body is known as

   a) Capacitance b) Conductance
   c) Inductance d) Resistance

2. Number of electric lines of force from 0.5 C if positive charge in a dielectric medium of constant 10 is

   a) 5.65×10^9 b) 1.13×10^{11}
   c) 9×10^9 d) 8.85×10^{-12}

3. n conducting wires of same dimensions but having resistivites 1,2,3.....n are connected in series. The equivalent resistivity of the combinations is

   a) $\frac{n(n+1)}{2}$ b) $\frac{n+1}{2}$
   c) $\frac{n+2}{2n}$ d) $\frac{2n}{n+1}$

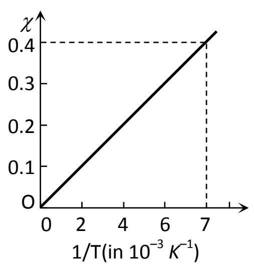
4. If nearly 10^5C liberate 1 g equivalent of aluminium, then the amount of aluminium (equivalent weight g) deposited through electrolysis in 20 min by a current of 50 A will be

   a) 0.09 g b) 0.6 g c) 5.4 g d) 10.8 g

5. Two particles of equal charges after being accelerated through the same potential difference enter a uniform transverse magnetic field and describe circular path of radii R_1 and R_2 respectively. Then the ratio of their masses (M_1/M_2) is

   a) $\frac{R_1}{R_2}$ b) $\left(\frac{R_1}{R_2}\right)^2$ c) $\frac{R_2}{R_1}$ d) $\left(\frac{R_2}{R_1}\right)^2$

6. The $\chi - 1/T$ graph for an alloy of paramagnetic nature is shown in fig. the curie constant is

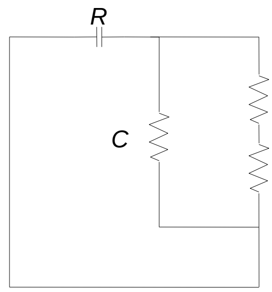


   a) $57\ K$ b) $2.8 \times 10^{-3} K$
   c) $570\ K$ d) $17.5 \times 10^{-3} K$

7. The device that does not work on the principle of mutual induction is

   a) Induction coil b) Motor
   c) Tesla coil d) Transformer

8. The time constant of the given circuit is

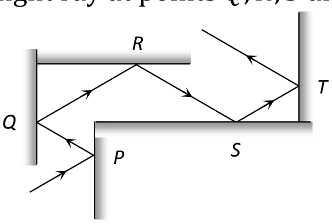


   a) $\frac{3RC}{5}$ b) $\frac{6RC}{5}$
   c) $\frac{5RC}{6}$ d) None of these

9. In an electromagnetic wave, the electric and magnetizing fields are 100 Vm^{-1} and 0.265 Am^{-1}. The maximum energy flow is

   a) 26.5 Wm^{-2} b) 36.5 Wm^{-2}
   c) 46.7 Wm^{-2} d) 765 Wm^{-2}

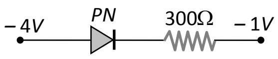
10. Following figure shows the multiple reflections of a light ray along a glass corridor where the walls are either parallel or perpendicular to one another. If the angle of incidence at point P is 30°, what are the angles of reflection of the light ray at points Q, R, S and T respectively

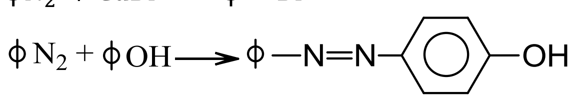


   a) 30°, 30°, 30°, 30° b) 30°, 60°, 30°, 60°
   c) 30°, 60°, 60°, 30° d) 60°, 60°, 60°, 60°

11. A light wave is incident normally over a slit of width $24 \times 10^{-5} cm$. The angular position of second dark fringe from the central maxima is 30°. What is the wavelength of light

   a) 6000 Å b) 5000 Å c) 3000 Å d) 1500 Å

12. The de-Broglie wavelength of the electron in the ground state of the hydrogen atom is (Radius of the first orbit of hydrogen atom =0.53Å)
a) 1.67 Å b) 3.33 Å c) 1.06 Å d) 0.53 Å
13. If λ_1 and $\lambda_2$are the wavelengths of the first members of the Lyman and Paschen series respectively, then $\lambda_1 : \lambda_2$is
a) 1:3 b) 1:30 c) 7:50 d) 7:108
14. The ratio of speed of an electron in ground state in Bohrs first orbit of hydrogen atom to velocity of light in air is
a) $\frac{e^2}{2\varepsilon_0 hc}$ b) $\frac{2e^2\varepsilon_0}{hc}$ c) $\frac{e^3}{2\varepsilon_0 hc}$ d) $\frac{2\varepsilon_0 hc}{e^2}$
15. What is the current in the circuit shown below


a) 0 A b) 10^{-2} A c) 1 A d) 0.10 A
16. The height of a TV antenna is 200 m. The population density is4000 km^{-2}. Find the population benefited
a) 3.2×10^8 b) 3.2×10^7
c) 3.2×10^6 d) 3.2×10^5
17. Sodium metal crystallises at room temperature in a body centred cubic lattice with a cell edge $a = 4.29$ Å. The radius of sodium atom is
a) 1.40 b) 2.65 c) 1.85 d) 2.15
18. Van't hoff factor of $Ca(NO_3)_2$ is
a) Benzoic acid is an organic solute
b) Benzoic acid has higher molar mass than benzene
c) Benzoic acid gets associated in benzene
d) Benzoic acid gets dissociated in benzene
19. For cell reaction,
$Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$
Cell representation is
a) Zn | Zn^{2+} || Cu^{2+} | Cu
b) Cu | Cu^{2+} || Zn^{2+} | Zn
c) Cu | Zn^{2+} || Zn | Cu^{2+}
d) Cu^{2+} | Zn || Zn^{2+} | Cu
20. In Arrhenius plot intercept is equal to
a) $-E_a/R$ b) $\ln A$
c) $\ln k$ d) $\log_{10} a$
21. A catalyst promoter
a) Increases the speed of the reaction
b) Activates the action of a catalyst
c) Starts a chemical reaction
d) None of the above
22. Which of the following is a carbonate ore?
a) Pyrolusite b) Diaspore
c) Cassiterite d) Malachite
23. The product obtained by heating $(NH_4)_2SO_4$ and KCNO is
a) Ammonia b) Ammonium cyanide
c) Urea d) Hydrocyanic acid
24. Which pair consists only acidic oxides?
a) CrO_3, Mn_2O_7 b) ZnO_2, Al_2O_3
c) CaO, ZnO d) Na_2O, Al_2O_3
25. Two isomers $X$and $Y$with the formula $Cr(H_2O)_5ClBr_2$ were taken for experiment on depression in freezing point. It was found that one mole of Xgave depression corresponding to 2 moles of particles and one mole of Ygave depression due to 3 moles of particles. The structural formula of $X$and Yrespectively, are
a) $[Cr(H_2O)_5Cl]Br_2$; $[Cr(H_2O)_4Br_2]Cl.H_2O$
b) $[Cr(H_2O)_5Cl]Br_2$; $[Cr(H_2O)_3ClBr_2].2H_2O]$
c) $[Cr(H_2O)_5Br]BrCl$; $[Cr(H_2O)_4ClBr]Br.H_2O$
d) $[Cr(H_2O)_4Br_2]ClH_2O$; $[Cr(H_2O)_5Cl]Br_2$
26. The number of possible enantiomeric pairs that can be produced during mono-chlorination of 2-methyl butane is
a) 3 b) 4 c) 1 d) 2
27. $C_6H_5 - CH = CHCHO \xrightarrow{X} C_6H_5CH = CHCH_2OH$
In the above sequence Xcan be
a) H_2/Ni b) $NaBH_4$
c) $K_2Cr_2O_7/H^+$ d) Both (a) and (b)
28. Correct order of reactivity of acid derivatives towards a nucleophile is
a) $RCOCl > (RCO)_2O > RCOOR > RCONH_2$
b) $RCOOR > RCOCl > RCONH_2 > (RCO)_2O$
c) $RCONH_2 > (RCO)_2O > RCOOR > RCOCl$
d) $(RCO)_2O > RCOCl > RCOOR > RCONH_2$
29. Which of the following reaction will not occur?
a) $\phi N_2^+ + CuBr \xrightarrow{HBr} \phi - Br$
b) 

c) $\phi N_2^+ \xrightarrow[HOH]{H_3PO_2} \phi H$
d) $\phi N_2^+ + I^- \rightarrow \phi - I$
30. Insulin production and its action in human body are responsible for the level of diabetes. This compound belongs to which of the following categories?
a) A co-enzyme b) A hormone
c) An enzyme d) An antibiotic
31. Bakelite is an example of

a) Elastomer
b) Fibre
c) Thermoplastic
d) Thermosetting polymer

32. Detergents obtained from LAB are biodegradable. LAB stands for
a) Laboratory tested raw material
b) Low anionic balance
c) Linear alkyl benzene
d) None of the above

33. If $y = \sin(\log_e x)$, then $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx}$ is equal to
a) $\sin(\log_e x)$ b) $\cos(\log_e x)$
c) y^2 d) $-y$

34. $\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right]$ is equal to
a) $\frac{71}{125}$ b) $\frac{74}{125}$ c) $\frac{3}{5}$ d) $\frac{1}{2}$

35. If A= $\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. The only correct statement about the matrix A is
a) A is a zero matrix
b) $A = (-1)I$, where I is a unit matrix
c) A^{-1} does not exist
d) $A^2 = I$

36. If $a, b. c$ are in AP, then the value of
$\begin{vmatrix} x+2 & x+3 & x+a \\ x+4 & x+5 & x+b \\ x+6 & x+7 & x+c \end{vmatrix}$ is
a) $x-(a+b+c)$ b) $9x^2+a+b+c$
c) $a+b+c$ d) 0

37. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to
a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) -13 d) $1/3$

38. Tangent is drawn to ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ (where$\theta \in (0, \pi/2)$). Then the value of θ such that sum of intercepts on axes made by this tangent is minimum, is
a) $\pi/3$ b) $\pi/6$ c) $\pi/8$ d) $\pi/4$

39. $\int \log 2x\, dx$ is equal to
a) $x\log 2x - \frac{x^2}{2} + c$ b) $x\log 2x - \frac{x}{2} + c$
c) $x^2\log 2x - \frac{x}{2} + c$ d) $x\log 2x - x + c$

40. If $f(x)$ and $g(x), x\epsilon R$ are continuous functions, then value of integral $\int_{-\pi/2}^{\pi/2}[\{f(x)+f(-x)\}\{g(x)-g(-x)\}]\, dx$ is
a) π b) $\frac{\pi}{2}$ c) 1 d) 0

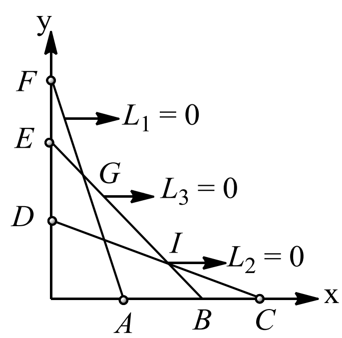
41. The area bounded by $y = \sin^{-1}x, x = \frac{1}{\sqrt{2}}$ and x-axis is
a) $\left(\frac{1}{\sqrt{2}}+1\right)$ sq units
b) $\left(1-\frac{1}{\sqrt{2}}\right)$ sq uints
c) $\frac{\pi}{4\sqrt{2}}$ sq units
d) $\left(\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)$ sq units

42. If $y = f(x)$ is the equation of the curve an its differential equation is given by $\frac{dy}{dx} = \frac{x+2}{y+3}$, then the equation of the curve, if it passes through (2, 2), is
a) $x^2 - y^2 + 4x - 6y + 4 = 0$
b) $x^2 - y^2 + 4x + 6y = 0$
c) $x^2 - y^2 - 4x - 6y = 0$
d) $x^2 - y^2 - 4x - 6y - 4 = 0$

43. If $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are any two non-collinear unit vectors and $\vec{\mathbf{a}}$ is any vector, then
$$(\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}) + \vec{\mathbf{b}}(\vec{\mathbf{a}}\cdot\vec{\mathbf{c}})\vec{\mathbf{c}} + \frac{\vec{\mathbf{a}}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})}{|\vec{\mathbf{b}}\times\vec{\mathbf{c}}|}\cdot(\vec{\mathbf{b}}\times\vec{\mathbf{c}})$$
is equal to
a) $\vec{\mathbf{0}}$ b) $\vec{\mathbf{a}}$ c) $\vec{\mathbf{b}}$ d) $\vec{\mathbf{c}}$

44. A equation of the plane passing through the points (3, 2, −1), (3, 4, 2) and (7, 0, 6) is $5x + 3y - 2z = \lambda$, where λ is
a) 23 b) 21 c) 19 d) 27

45. The feasible region for the following constraints $L_1 \leq 0$ $L_2 \geq 0, L_3 = 0, x \geq 0, y \geq 0$ in the diagram shown is



a) Area DHF b) Area AHC
c) Line segment EG d) Line segment GI

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: ANSWER KEY :

1)	a	2)	a	3)	a	4)	c	5)	b	6)	a	7)	c
8)	b	9)	a	10)	c	11)	a	12)	b	13)	d	14)	a
15)	a	16)	b	17)	c	18)	c	19)	a	20)	b	21)	b
22)	d	23)	c	24)	a	25)	d	26)	d	27)	b	28)	a
29)	b	30)	b	31)	d	32)	c	33)	d	34)	a	35)	d
36)	d	37)	b	38)	b	39)	d	40)	d	41)	d	42)	a
43)	b	44)	a	45)	c								

: HINTS AND SOLUTIONS :

2 **(a)**

$$\phi_E = \frac{q}{K\varepsilon_0} = \frac{0.5}{10 \times 8.85 \times 10^{12}} = 5.65 \times 10^9$$

3 **(a)**

Resistivity of combination

$= (1 + 2 + 3 + \cdots + n)$

$= \dfrac{n(n+1)}{2}$

4 **(c)**

$m = zit$

$9 = z \times 10^5 \Rightarrow z = \dfrac{9}{10^5}\,\text{g C}^{-1}$

$\therefore m = zit = 9 \times 10^{-5} \times 50 \times (20 \times 60)$ =5.4 g.

5 **(b)**

Radius of circular path

$R = \frac{mv}{qB}$

But $mv = \sqrt{2mqV}$

$\therefore R = \dfrac{\sqrt{2mqV}}{qB}$ or $R \propto \sqrt{m}$

or $\frac{R_1^2}{R_2^2} = \frac{M_1}{M_2}$

or $\frac{M_1}{M_2} = \frac{R_1^2}{R_2^2} = \left(\frac{R_1}{R_2}\right)^2$

6 **(a)**

$$X = C \times \frac{1}{T} = \frac{0.4}{7 \times 10^{-3}} = 57K$$

8 **(b)**

Time constant of $R - C$ circuit is $\tau = RC$

Here effective resistance of the circuit

$= \frac{2R \times 3R}{2R + 3R} = \frac{6R}{5}$

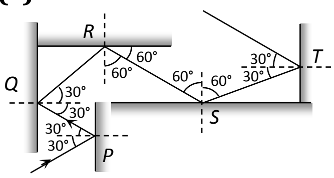
$\therefore \quad \tau = \frac{6R}{5} \times C = \frac{6RC}{5}$

9 **(a)**

Here, amplitude of electric filed, $E_0 = 100\ \text{Vm}^{-1}$; amplitude of magnetic field, $B_0 = 0.265\ \text{Am}^{-1}$. We know that the maximum rate of energy flow

$S = E_0 \times B_0 = 100 \times 0.265 = 26.5\ \text{Wm}^{-2}$

10 **(c)**



11 **(a)**

For second dark fringe $d \sin\theta = 2\lambda$

$\Rightarrow 24 \times 10^{-5} \times 10^{-2} \times \sin 30 = 2\lambda$

$\Rightarrow \lambda = 6 \times 10^{-7} m = 6000$ Å

12 **(b)**

According to Bohr's quantisation of angular momentum

$$mvr = \frac{nh}{2\pi}$$

Or $\quad \frac{h}{mv} = \frac{2\pi r}{n} \quad ..(i)$

de-Broglie wavelength

$$\lambda = \frac{h}{mv} \quad ..(ii)$$

From Eqs. (i) and (ii), we get

Wavelength $\lambda = \frac{2\pi r}{n}$

$= \frac{2 \times \pi \times 0.53\ \text{Å}}{1} = 3.33$Å

13 **(d)**
For first line of Lyman series,
n_1=1 and n_2=2
$\therefore \quad \frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4}$
For first line of Paschen Series
n_1=3 and n_2=4
$\therefore \quad \frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144}$
$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{7R}{144} \times \frac{4}{3R} = \frac{7}{108}$

14 **(a)**
Speed of electron in n^{th} orbit (in CGS)
$v_n = \frac{2\pi Ze^2}{nh} (k = 1)$
For first orbit of H_1; $n = 1$ and $Z = 1$
So $v = \frac{2\pi e^2}{h} \Rightarrow \frac{v}{c} = \frac{2\pi e^2}{hc}$

15 **(a)**
The potential of P-side is more negative that of N-side, hence diode is in reverse biasing. In reverse biasing it acts as open circuit, hence no current flows

16 **(b)**
$d = \sqrt{2hR}$;
Population converted= $\pi d^2 \times$ population density
$= \pi(2Rh) \times \rho$
$= \frac{22}{7} \times 2 \times 6400 \times 0.2 \times 4000$
$= 3.2 \times 10^7$

17 **(c)**
For a body centred cubic lattice radius, (r)
$$= \frac{\sqrt{3}}{4} a = 0.433a$$
Therefore, radius of $Na^+ = 0.433 \times 4.29 = 1.8575$

18 **(c)**
Benzoic acid in benzene exists as a dimer. So, number of molecules decreases and hence, osmotic pressure decreases.

19 **(a)**
Cell representation is done as follows
Anode | Anodic electrolyte || cathodic electrolyte | cathode
(i) Oxidation is loss of electron and it takes place at anode. Reduction is gain of electron and it takes place at cathode.

$\therefore$ For cell reaction,
$Zn + Cu^{2+} \longrightarrow Zn^{2+} + Cu$
Zn is anode and Cu is cathode.

$\therefore$ Cell representation is
$Zn \mid Zn^{2+} \parallel Cu^{2+} \mid Cu$

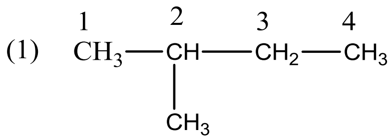
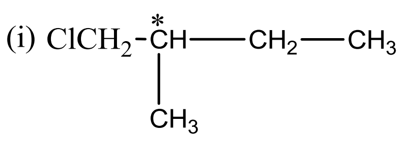
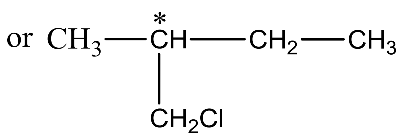
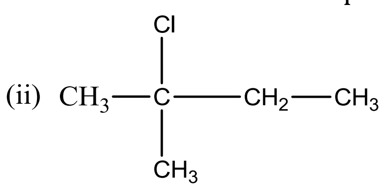
21 **(b)**
A promoter is not a catalyst but it activates the action of a catalyst

22 **(d)**
Pyrolusite – MnO_2
Malachite – $CuCO_3.Cu(OH)_2$
Diaspore –$Al_2O_3.H_2O$
Cassiterite –SnO_2

23 **(c)**
$(NH_4)_2SO_4 + KCNO \rightarrow NH_4CNO + K_2SO_4$
↓
NH_2CONH_2
urea

24 **(a)**
CrO_3 and Mn_2O_7 are acidic oxide. Since, they react with water and form the acids.
e.g., $CrO_3 + H_2O \longrightarrow H_2CrO_4$
chromic acid
$Mn_2O_7 + H_2O \longrightarrow 2HMnO_4$
permanganic acid

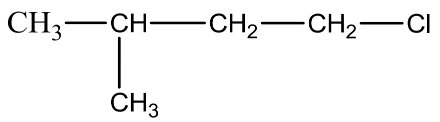
25 **(d)**
One mole of X gave depression corresponding to 2 moles of particles, *i.e.*, on ionisation X gives 2 moles of ions , thus it contains only 1 ion outside the coordination sphere and its structural formula is $[Cr(H_2O)_4Br_2]Cl.H_2O$ while Y gives 3 moles of ions, thus it contains two ions outside the coordination sphere and its structural formula is $[Cr(H_2O)_5\ Cl]Br_2 2$

26 **(d)**

Its monochloro derivatives are follows


It will exist as enantiomers pair d and l-forms


no asymmetric C atom

Cl
|
(iii) $CH_3 - CH - CH - CH_3$
|
CH_3

It will exist as enantiomeric pair (d-and l- forms)

(iv) 

No asymmetric carbon atom

Hence, only two enantiomeric pairs will be obtained by the monochlorination of 2-methylbutane.

27 **(b)**

$NaBH_4$ and $LiAlH_4$ attacks only carbonyl group and reduce it into alcohol group. They do not attack on double bond.

$$C_6H_5 - CH = CHCHO \xrightarrow{NaBH_4} C_6H_5 - CH = CH.CH_2OH$$

cinnamic aldehyde cinnamic alcohol

28 **(a)**

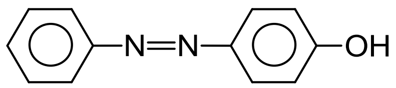
The relative reactivity of the acid derivatives towards nucleophilic acyl substitution reaction follow the order :

$RCOCl > (RCO)_2O > RCOOR > RCONH_2$

The ease with which these leaving groups depart decreases in the order: $Cl^- > RCOO^- > RCO^- > NH_2^-$. Consequently the relative reactivities of all these acid derivatives decreases in the order : acid chloride > anhydride > ester > amide

29 **(b)**

$\phi N_2^+ + \phi OH \rightarrow \phi N_2\ \phi OH$

or 

coupling product

30 **(b)**

Insulin is proteinaceous hormone. It is secreted by pancreas and controls the metabolism of glucose and maintains glucose level in the blood

31 **(d)**

Bakelite, due to presence of extensive crosslinking, is an example of thermosetting polymer

33 **(d)**

Given, $y = \sin\ (\log_e x)$ ...(i)

$\Rightarrow\ \frac{dy}{dx} = \cos(\log_e x).\frac{1}{x}$ ...(ii)

$\Rightarrow\ \frac{d^2y}{dx^2} = \frac{-x.\sin(\log_e x).\frac{1}{x} = \cos(\log_e x).1}{x^2}$

$= \frac{-\sin\ (\log_e x) - \cos\ (\log_e x)}{x^2}$

$x^2\frac{d^2y}{dx^2} = -\sin(\log_e x) - x\frac{dy}{dx}$ [using Eq. (ii)]

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$ [using Eq. (i)]

34 **(a)**

$$\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right] = \sin\left[\sin^{-1}\left\{3\left(\frac{1}{5}\right) - 4\left(\frac{1}{5}\right)^3\right\}\right]$$

$$= \frac{3}{5} - \frac{4}{125} = \frac{71}{125}$$

35 **(d)**

(a) It is clear that A is not a zero matrix.

(b) $(-1)I = -1\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} = \begin{bmatrix}-1 & 0 & 0\\0 & -1 & 0\\0 & 0 & -1\end{bmatrix} \neq A$

$ie,\quad (-1)I \neq A$

(c) $|A| = 0\begin{vmatrix}-1 & 0\\0 & 0\end{vmatrix} - 0\begin{vmatrix}0 & 0\\-1 & 0\end{vmatrix} - 1\begin{vmatrix}0 & -1\\-1 & 0\end{vmatrix}$

$= 0 - 0 - 1(-1) = 1$

Since,$|A| \neq 0$ so A^{-1} exists.

(d) $A^2 = \begin{bmatrix}0 & 0 & -1\\0 & -1 & 0\\-1 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & -1\\0 & -1 & 0\\-1 & 0 & 0\end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \Rightarrow A^2 = I$

36 **(d)**

Let $\Delta = \begin{vmatrix}x+2 & x+3 & x+a\\x+4 & x+5 & x+b\\x+6 & x+7 & x+c\end{vmatrix}$

Applying $C_2 \rightarrow C_2 - C_1$,we get

$\Delta = \begin{vmatrix}x+2 & 1 & x+a\\x+4 & 1 & x+b\\x+6 & 1 & x+c\end{vmatrix}$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$\Rightarrow\ \Delta = \begin{vmatrix}x+2 & 1 & x+a\\2 & 0 & b-a\\4 & 0 & c-a\end{vmatrix}$

$= -1(2c - 2a - 4b + 4a)$

$\Rightarrow\ \Delta = 2(2b - c - a)$ ...(i)

Since, a, b, c are in AP.

$\therefore\ b = \frac{a+c}{2}$

$\therefore\ \Delta = 2(a + c - c - a)$

$= 0$ [from Eq. (i)]

37 **(b)**

$f'(1) = \lim_{h\rightarrow 0}\frac{f(1+h)-f(1)}{h}$

$= \lim_{h\rightarrow 0}\frac{\frac{1+h-1}{2(1+h)^2-7(1+h)+5} - \left(\frac{1}{3}\right)}{h}$

$$= \lim_{h\to 0} \frac{\left(\frac{1}{2h-3}+\frac{1}{3}\right)}{h} = \lim_{h\to 0}\left(\frac{2h}{3h(2h-3)}\right) = -\frac{2}{9}$$

38 **(b)**

Equation of tangent at $(3\sqrt{3}, \cos\theta, \sin\theta)$ is

$$\frac{x\cos\theta}{3\sqrt{3}} + \frac{y\sin\theta}{1} = 1$$

Thus, sum of intercepts= $\left(3\sqrt{3}\sec\theta + \operatorname{cosec}\theta\right) = f(\theta)$ [say]

$$\Rightarrow \quad f'(\theta) = \frac{3\sqrt{3}\sin^3\theta - \cos^3\theta}{\sin^2\theta\cos^2\theta}$$

Put $f'(\theta) = 0$

$$\therefore \quad \sin^3\theta = \frac{1}{3^{3/2}}\cos^3\theta$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad \theta = \frac{\pi}{6}$$

Also, for $0 < \theta < \frac{\pi}{6}, \frac{dz}{d\theta} < 0$ and for $\frac{\pi}{6} < \theta < \frac{\pi}{2}, \frac{dz}{d\theta} > 0$

$\therefore$Minimum at $\theta = \frac{\pi}{6}$

39 **(d)**

Let $I = \int 1 . \log 2x \, dx$

$$\Rightarrow \quad I = x\log 2x - \int \frac{1}{2x} . 2 . x \, dx$$

$$\Rightarrow \quad I = x\log 2x - x + c$$

40 **(d)**

Let $I = \int_{-\pi/2}^{\pi/2} \{f(x)+f(-x)\}\{g(x)-g(-x)\} \, dx$

Again, let

$h(x) = \{f(x)+f(-x)\}\{g(x)-g(-x)\}$

$\Rightarrow h(-x) = \{f(-x)+f(x)\}\{g(-x)-g(x)\}$

$\Rightarrow h(-x) = -h(x)$

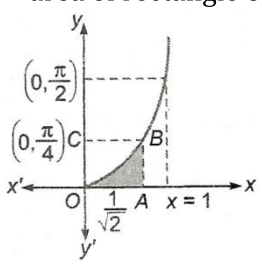
Hence, $h(x)$ is an odd function

$\therefore \quad I = 0$

41 **(d)**

Required area

= area of rectangle $OABC$ − area of curve $OBCO$



$$= \frac{\pi}{4\sqrt{2}} - \int_0^{\frac{\pi}{4}} \sin y \, dy$$

$$= \frac{\pi}{4\sqrt{2}} + [\cos y]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{\pi}{4\sqrt{2}} + \left(\frac{1}{\sqrt{2}} - 1\right)\right] \text{ sq unit}$$

42 **(a)**

The given equation is

$(y+3)dy = (x+2)dx$

$$\Rightarrow \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + c$$

Since, it passes through (2, 2).

$\therefore 2 + 6 = 2 + 4 + c \ \Rightarrow c = 2$

$$\therefore \frac{y^2}{2} + 3y = \frac{x^2}{2} + 2x + 2$$

$\Rightarrow y^2 + 6y = x^2 + 4x + 4$

$\Rightarrow x^2 + 4x - y^2 - 6y + 4 = 0$

43 **(b)**

Let $\vec{\mathbf{b}} = \hat{\mathbf{i}}$ and $\vec{\mathbf{c}} = \hat{\mathbf{j}}$

$\therefore |\vec{\mathbf{b}} \times \vec{\mathbf{c}}| = |\hat{\mathbf{k}}| = 1$

Let $\vec{\mathbf{a}} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$

Now, $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = \vec{\mathbf{a}}\,\hat{\mathbf{i}} = a_1, \vec{\mathbf{a}} \cdot \vec{\mathbf{c}} = \vec{\mathbf{a}} \cdot \hat{\mathbf{j}} = a_2$

and $\vec{\mathbf{a}} \cdot \dfrac{\vec{\mathbf{b}} \times \vec{\mathbf{c}}}{|\vec{\mathbf{b}} \times \vec{\mathbf{c}}|} = \vec{\mathbf{a}} \cdot \hat{\mathbf{k}} = a_3$

$$\therefore \left(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}\right)\vec{\mathbf{b}} + (\vec{\mathbf{a}} \cdot \vec{\mathbf{c}})\vec{\mathbf{c}} + \frac{\vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})}{|\vec{\mathbf{b}} \times \vec{\mathbf{c}}|} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}})$$

$= a_1\vec{\mathbf{b}} + a_2\vec{\mathbf{c}} + a_3\left(\vec{\mathbf{b}} \times \vec{\mathbf{c}}\right)$

$= a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}} = \vec{\mathbf{a}}$

44 **(a)**

Equation of plane through $(3, 2, -1)$ is

$a(x-3) + b(y-2) + c(z+1) = 0$ ...(i)

Also, (3, 4, 2) and (7, 0, 6) lie on Eq. (i), then

$0 \cdot a + 2b + 3c = 0$ ...(ii)

And $4a - 2b + 7c = 0$ ...(iii)

On eliminating a, b, c from Eqs. (i), (ii) and (iii), we get

$$\begin{vmatrix} x-3 & y-2 & z+1 \\ 0 & 2 & 3 \\ 4 & -2 & 7 \end{vmatrix} = 0$$

We get, $5x + 3y - 2z = 23$

$\therefore \ \lambda = 23$

45 **(c)**

In the given figure the feasible region for given constraints is the line segment EG